NCERT Solutions for Class 12 Maths Chapter 9 – Differential Equations

Exercise 9.1

Determine order and degree (if defined) of differential equations given in Exercises 1 to 10.

1.

$$\frac{d^4y}{dx^4} + \sin(y''') = 0$$

Ans - Given equation can be written as,

$y'''' + \sin(y''') = 0$

Highest order derivative present in the differential equation is y'''', so its order is four.

∴ Given differential equation is not a polynomial equation in its derivatives, and hence its degree is unknown.

2.

$$y' + 5y = 0$$

Ans - Highest order derivative present in the differential equation is y', so its order is one.

∴ Given differential equation is a polynomial equation in its derivatives, and hence its degree is one.

3.

$$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$$

Ans - Highest order derivative present in the differential equation is $\frac{d^2s}{dt^2}$, so its order is two.

$\therefore$ Given differential equation is a polynomial equation in $\frac{d^2s}{dt^2}$ and $\frac{ds}{dt}$, and hence its degree is one.

4.

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$$

Ans - Highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$, so its order is two.

$\therefore$ Given differential equation is not a polynomial equation in its derivatives, and hence its degree is unknown.

5.

$$\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$$

Ans - Highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$, so its order is two.

$\therefore$ Given differential equation is a polynomial equation in $\frac{d^2y}{dx^2}$, and hence its degree is one.

6.

$$(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$$

Ans - Highest order derivative present in the differential equation is y''', so its order is three.

$\therefore$ Given differential equation is a polynomial equation in y''', y'' and y'. Power of y''' is 2, hence its degree is two.

7.

$$y''' + 2y'' + y' = 0$$

Ans - Highest order derivative present in the differential equation is y''', so its order is three.

$\therefore$ Given differential equation is a polynomial equation in y''', y'' and y'. Power of y''' is 1, hence its degree is one.

8.

$\boldsymbol{y' + y = e^x}$

Ans - Given equation can be written as,

$y' + y - e^x = 0$

Highest order derivative present in the differential equation is y', so its order is one.

$\therefore$ Given differential equation is a polynomial equation in y'. Power of y' is 1, hence its degree is one.

9.

$\boldsymbol{y'' + (y')^2 + 2y = 0}$

Ans - Highest order derivative present in the differential equation is y'', so its order is two.

$\therefore$ Given differential equation is a polynomial equation in y'' and y'. Power of y'' is 1, hence its degree is one.

10.

$\boldsymbol{y'' + 2y' + \sin y = 0}$

Ans - Highest order derivative present in the differential equation is y'', so its order is two.

$\therefore$ Given differential equation is a polynomial equation in y'' and y'. Power of y'' is 1, hence its degree is one.

11.

The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + sin\left(\frac{dy}{dx}\right) + 1 = 0 \text{ is}$$

(A) 3 (B) 2 (C) 1 (D) not defined

Ans - Given equation can be rewritten as,

$(y'')^3 + (y')^2 + sin(y') + 1 = 0$

Above differential equation is not a polynomial equation. So its degree is not defined.

Hence correct answer is option (D).

12.

The order of the differential equation

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0 \text{ is}$$

(A) 2 (B) 1 (C) 0 (D) not defined

Ans - Highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$, so its order is two.

Hence correct answer is option (A).

Exercise 9.2

In each of the Exercises 1 to 10 verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

1.

$y = e^x + 1 : y'' - y' = 0$

Ans - Given equation is $y = e^x + 1$

Differentiating equation on both the sides of w.r.t. x, the equation becomes,

$$\frac{dy}{dx} = \frac{d}{dx}(e^x + 1)$$

Derivative of e^x is e^x and the derivative of a constant, $\frac{d}{dx}(a) = 1$, where a is constant.

$$\Rightarrow \frac{dy}{dx} = e^x$$

$\Rightarrow y' = e^x \quad \dots\dots(1)$

Now differentiate the obtained equation again w.r.t. x,

$$\Rightarrow \frac{d}{dx}(y') = \frac{d}{dx}(e^x)$$

$= y'' = e^x \quad \dots\dots(2)$

Given LHS $= y'' - y'$

From (1) and (2) we get,

$y'' - y' = e^x - e^x = 0$

$\therefore$ LHS = RHS

Hence, provided function is the differential equation's solution.

2.

$\boldsymbol{y = x^2 + 2x + C: y' - 2x - 2 = 0}$

Ans - Given equation is $y = x^2 + 2x + C$

Differentiating equation on both sides w.r.t. x, the equation becomes.

$$y' = \frac{d}{dx}(x^2 + 2x + C)$$

$$\Rightarrow y' = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x) + \frac{d}{dx}(C)$$

$$\Rightarrow y' = 2x + 2$$

Given LHS $= y' - 2x - 2$

$$\Rightarrow 2x + 2 - 2x - 2 = 0$$

$\therefore$ LHS = RHS

Hence, provided function is the differential equation's solution.

3.

$$\mathbf{y = \cos x + C : y' + \sin x = 0}$$

Ans $-$ Given equation is $y = \cos x + \mathrm{C}$

Differentiating equation on both sides w.r.t. x, the equation becomes,

$$y' = \frac{d}{dx}(\cos x + C)$$

$$\Rightarrow y' = \frac{d}{dx}(\cos x) + \frac{d}{dx}(C)$$

$$\Rightarrow y' = -\sin x$$

Given LHS $= y' + \sin x = 0$

$$\Rightarrow -\sin x + \sin x = 0$$

$\therefore$ LHS = RHS

Hence, provided function is the differential equation's solution.

4.

$$\mathbf{y = \sqrt{1 + x^2} : y' = \frac{xy}{1 + x^2}}$$

Ans - Given equation is $y = \sqrt{1+x^2}$

Differentiating equation on both sides w.r.t.. x,

$$y' = \frac{d}{dx}\left(\sqrt{1+x^2}\right)$$

Derivative of $\sqrt{x}$ is $\frac{1}{2\sqrt{x}}$ and derivative of x^n $\frac{d}{dx}(x^n) = nx^{n-1}$.

$$\Rightarrow y' = \frac{1}{2\sqrt{1+x^2}} \cdot \frac{d}{dx}(1+x^2)$$

$$\Rightarrow y' = \frac{2x}{2\sqrt{1+x^2}}$$

$$\Rightarrow y' = \frac{x}{\sqrt{1+x^2}}$$

Now, multiply numerator and denominator of RHS of the above equation by $\sqrt{1+x^2}$.

$$\Rightarrow y' = \frac{x}{\sqrt{1+x^2}}\left(\frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow y' = \frac{x}{1+x^2}.\sqrt{1+x^2}$$

Substitute y for $\sqrt{1+x^2}$ in the above equation

$$\Rightarrow y' = \frac{x}{1+x^2}.y$$

$$\Rightarrow y' = \frac{xy}{1+x^2}$$

$\therefore$ LHS = RHS

Hence, provided function is the differential equation's solution.

5.

$\mathbf{y = Ax: xy' = y(x \neq 0)}$

Ans - Given equation is $y = Ax$

Differentiating equation on both sides of w.r.t. x,

$y' = \frac{d}{dx}(Ax)$

$\Rightarrow y' = A\frac{d}{dx}(x)$

$\Rightarrow y' = A(1) = A$

Given LHS $= xy'$

$\Rightarrow xy' = xA$

$\Rightarrow y' = A$

$\Rightarrow y = Ax$

$\therefore$ LHS = RHS

Hence, provided function is the differential equation's solution.

6.

$\mathbf{y = x\sin x : xy' = y + x\sqrt{x^2 - y^2}(x \neq 0 \text{ and } x > y \text{ or } x < -y).}$

Ans - Given equation is $y = x\sin x$.

Differentiating equation on both sides w.r.t. x,

$y' = \frac{d}{dx}(x\sin x)$

The product rule of differential equations says that,

$\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$

$\Rightarrow y' = x\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(x)$

$\Rightarrow y' = x\cos x + \sin x$

Now, consider the LHS of the equation $xy' = y + x\sqrt{x^2 - y^2}$

Substitute the obtained value for y' in the LHS of the above equation.

$\Rightarrow \mathrm{xy}' = \mathrm{x}(\mathrm{x}\cos x + \sin x)$

$\Rightarrow \mathrm{xy}' = \mathrm{x}^2(\mathrm{x}\cos x + \sin x)$

It is known that $\sin^2 x + \cos^2 x = 1$ thus, $\cos x = \sqrt{1 - \sin^2 x}$ and $\sin x = \frac{y}{x} =$ as $y = x\sin x$. Thus,

$$\Rightarrow xy' = x^2\sqrt{1 - \sin^2 x} + y$$

$$\Rightarrow xy' = x^2\sqrt{1 - \left(\frac{y}{x}\right)^2} + y$$

$$\Rightarrow xy' = \frac{x^2}{x}\sqrt{x^2 - y^2} + y$$

$$\Rightarrow xy' = x\sqrt{x^2 - y^2} + y$$

$\therefore$ LHS = RHS

Hence, provided function is the differential equation's solution.

7.

$$\mathbf{xy = \log y + C: y' = \frac{y^2}{1 - xy}(xy \neq 1).}$$

Ans - Given equation is $xy = \log y + C$

Differentiating equation on both sides w.r.t. x,

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y)$$

The product rule of differential equations says that,

$$\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$$

$$\Rightarrow y\frac{d}{dx}(x) + x\frac{d}{dx}(y) = \frac{d}{dx}(\log y)$$

$$\Rightarrow y(1) + xy' = \frac{1}{y}y'$$

$\Rightarrow y^2 + xyy' = y'$

We know $y' = \frac{y^2}{1 - xy}(xy \neq 1)$

Substituting the value of y' in LHS,

$\Rightarrow y' = y^2 + xyy'$

$\Rightarrow y' - xyy' = y^2$

$\Rightarrow y'(1 - xy) = y^2$

$\Rightarrow y' = \frac{y^2}{(1 - xy)}$

$\therefore$ LHS = RHS

Hence, provided function is the differential equation's solution.

8.

$\mathbf{y - \cos y = x: (y \sin y + \cos y + x)y' = 1}$

Ans - Given equation is $y - \cos y$

Differentiating equation on both sides w.r.t. x,

$\frac{d}{dx} y - \frac{d}{dx} \cos y = \frac{d}{dx} x$

$\Rightarrow y' \sin y \cdot y' = 1$

$\Rightarrow y'(1 + \sin y) = 1$

$\Rightarrow y' = \frac{1}{1 + \sin y}$

We know the equation $(y\sin y + \cos y + x)y' = 1$.

Substitute obtained value for y' in LHS of above equation.

$\Rightarrow (y\sin y + \cos y + x)y'$

$$= (y\sin y + \cos y + y - \cos y) \times \frac{1}{1 + \sin y}$$

$\Rightarrow y(1 + \sin y).\frac{1}{\sin y} = y$

$\therefore$ LHS = RHS

Hence, provided function is the differential equation's solution.

9.

$\mathbf{x = y = \tan^{-1} y : y^2y' + y^2 + 1 = 0}$

Ans - Given equation is $x + y = \tan^{-1}y$.

Differentiating equation on both sides w.r.t. x,

$\frac{d}{dx}(x + y) = \frac{d}{dx}(\tan^{-1}y)$

$\Rightarrow 1 + y' = \left[\frac{1}{1 + y^2}\right] y'$

$\Rightarrow y'\left[\frac{1}{1 + y^2} - 1\right] = 1$

$\Rightarrow y'\left[\frac{1 - (1 + y^2)}{1 + y^2}\right] = 1$

$\Rightarrow y'\left[\frac{-y^2}{1 + y^2}\right] = 1$

$\Rightarrow y' = \frac{(-1 + y^2)}{y^2}$

We know the equation $y^2y' + y^2 + 1 = 0$

Substitute obtained value of y' in LHS of above equation.

$$\Rightarrow y^2\left[\frac{-(1+y^2)}{y^2}\right]+y^2+1=0$$

$$\Rightarrow -1-y^2+y^2+1=0$$

$$\Rightarrow y^2y'+y^2+1=0$$

$\therefore$ LHS = RHS

Hence, provided function is the differential equation's solution.

10.

$$\mathbf{y=\sqrt{a^2-x^2}x\in(-a,a):x+y\frac{dy}{dx}=0(y\neq 0).}$$

Ans - Given equation is $y=\sqrt{a^2-x^2}$

Differentiating equation on both sides w.r.t. x,

$$y'=\frac{d}{dx}\left(\sqrt{a^2-x^2}\right)$$

$$\Rightarrow y'=\frac{1}{2\sqrt{a^2-x^2}}\cdot\frac{d}{dx}(a^2-x^2)$$

$$\Rightarrow y'=\frac{-2x}{2\sqrt{a^2-x^2}}$$

$$\Rightarrow y'=\frac{-x}{\sqrt{a^2-x^2}}$$

We know the equation, $x+y\frac{dy}{dx}=0$.

Substitute y for $\frac{-x}{\sqrt{a^2-x^2}}$ and $\sqrt{a^2-x^2}$ for y in the LHS of the above equation,

$$\Rightarrow x+y\frac{dy}{dx}=x+\sqrt{a^2-x^2}\cdot\frac{-x}{\sqrt{a^2-x^2}}$$

$$\Rightarrow y'=x-x$$

$$\Rightarrow y'=0$$

$\therefore$ LHS = RHS

Hence, provided function is the differential equation's solution.

11.

The number of arbitrary constants in the general solution of a differential equation of fourth order are:
(A)0 (B)2 (C)3 (D)4

Ans – The number of constants in the general solution of a differential equation of order n is equal to the order of the differential equation. Hence, the general equation of a fourth-order differential equation contains four constants. Therefore, option D is the correct answer.

12.

The number of arbitrary constants in the particular solution of a differential equation of third order are:
(A)3 (B)2 (C)1 (D)0

Ans – When the general solution's arbitrary constant takes on a unique value, it becomes the problem's particular solution. Applying boundary conditions yields the particular solution to a differential equation. It is well known that no arbitrary constants are found in any given solution of a differential equation. Hence, the correct answer is option D.

Exercise 9.3

1.

Find the general solution for $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$

Ans - Given differential equation is $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

Use trigonometric half-angle identities to simplify

$$\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \tan^2\frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx} = \sec^2\frac{x}{2} - 1$$

Separate the differentials and integrate

$$\int dy = \int \left(\sec^2\frac{x}{2} - 1\right) dx$$

$$\Rightarrow y = \int \sec^2\frac{x}{2} dx - \int dx$$

$$\Rightarrow y = 2\tan\frac{x}{2} - x + C$$

∴ General solution of given differential equation is

$$y = 2\tan\frac{x}{2} - x + C$$

2.

Find the general solution for $\frac{dy}{dx} = \sqrt{4-y^2}(-2\ y\ 2)$

Ans - Given differential equation is $\frac{dy}{dx} + \mathbf{y} = 1(\mathbf{y} \neq 1)$

Simplify the expression:

$$\frac{dy}{dx} + y = 1$$

$$\Rightarrow \frac{dy}{dx} = 1 - y$$

$$\Rightarrow \frac{dy}{1 - y} = dx$$

Use standard integration

$$\int \frac{dy}{1 - y} = \int dx$$

$$\Rightarrow -\log(1 - y) = x + C$$

$$\Rightarrow \log(1 - y) = -(x + C)$$

$$\Rightarrow 1 - y = e^{-(x+C)}$$

$$y = 1 - Ae^{-x}(A = e^{-C})$$

∴ General solution of given differential equation is $y = 1 - Ae^{-x}$.

3.

Find the general solution for $\mathbf{\frac{dy}{dx} + y = 1(y \neq 1)}$

Ans - Given differential equation is $\frac{dy}{dx} + \mathbf{y} = 1(\mathbf{y} \neq 1)$

Simplify the expression:

$$\frac{dy}{dx} + y = 1$$

$$\Rightarrow \frac{dy}{dx} = 1 - y$$

$$\Rightarrow \frac{dy}{1-y} = dx$$

Use standard integration

$$\int \frac{dy}{1-y} = \int dx$$

$$\Rightarrow -\log(1-y) = x + C$$

$$\Rightarrow \log(1-y) = -(x+C)$$

$$\Rightarrow 1 - y = e^{-(x+C)}$$

$$y = 1 - Ae^{-x}(A = e^{-C})$$

$\therefore$ General solution of given differential equation is $y = 1 - Ae^{-x}$.

4.

Find the general solution for $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

Ans - Given differential equation is

$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

Divide both side by tan x tan y

$$\frac{\sec^2 x \tan y dx + \sec^2 y \tan x dy}{\tan x \tan y} = 0$$

$$\Rightarrow \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

Integrate both side:

$$\int \frac{\sec^2 x}{\tan x} dx + \int \frac{\sec^2 y}{\tan y} dy = 0$$

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy = -\int \frac{\sec^2 x}{\tan x} dx \qquad \dots(1)$$

Use a substitution method for integration. Substitute
$tan\ x = u$

For integral on RHS:

$\Rightarrow \tan x = u$

$\Rightarrow \sec^2 x dx = du$

$$\Rightarrow \int \frac{\sec^2 x}{\tan x} dx = \int \frac{du}{u}$$

$$\Rightarrow \int \frac{\sec^2 x}{\tan x} dx = \log u$$

$$\Rightarrow \int \frac{\sec^2 x}{\tan x} dx = \log(\tan x)$$

Thus evaluating result form (1)

$\Rightarrow \log(\tan y) = -\log(\tan x) + \log(C)$

$$\Rightarrow \log(\tan y) = \log\left(\frac{C}{\tan x}\right)$$

$\Rightarrow \tan x \tan y = C$

∴ General solution of given differential equation is
$\tan x \tan y = C$

5.

Find the general solution for
$$(\mathbf{e^x + e^{-x}})\mathbf{dy} - (\mathbf{e^x - e^{-x}})\mathbf{dx} = \mathbf{0}$$

Ans - Given differential equation is

$(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0.$

$$dy = \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right] dx$$

Integrate both side

$$\int dy = \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right] dx \quad \ldots\ldots(1)$$

Using substitution method for integration. Substitute $e^x + e^{-x} = t$:

For integral on RHS

$\Rightarrow e^x + e^{-x} = t$

$\Rightarrow (e^x - e^{-x})dx = dt$

$$\Rightarrow \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right] dx = \int \frac{dt}{t} \text{ s}$$

$$\Rightarrow \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right] dx = \ln t + C$$

$$\Rightarrow \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right] dx = \log(e^x + e^{-x}) + C$$

Thus, evaluating result form (1):

$y = \log(e^x + e^{-x}) + C$

∴ General solution of given differential equation is $y = \log(e^x + e^{-x}) + C$.

6.

Find the general solution for $\frac{\mathbf{dy}}{\mathbf{dx}} = (\mathbf{1 + x^2})(\mathbf{1 + y^2})$

Ans - Given differential equation is

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

$$\frac{dy}{1 + y^2} = (1 + x^2)dx$$

Integrate both side:

$$\int \frac{dy}{1 + y^2} = \int (1 + x^2)dx$$

Use standard integration:

$$\tan^{-1} y = \int (1 + x^2)dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C$$

∴ General solution of given differential equation is $\tan^{-1} y = x + \frac{x^3}{3} + C$.

7.

Find the general solution for $y \log y \, dx - xdy = 0$

Ans - Given differential equation is:

$y \log y \, dx - xdy = 0.$

Simplify the expression

$y \log y \, dx - xdy = 0$

$$\Rightarrow \frac{dx}{x} = \frac{dy}{y \log y}$$

Integrate both side

$$\int \frac{dy}{y \log y} = \int \frac{dx}{x} \quad \ldots \ldots (1)$$

Use substitution method for integration on LHS. Substitute $\log y = t$.

$$\Rightarrow \frac{1}{y} dy = dt$$

$$\Rightarrow \int \frac{dy}{y\log y} = \int \frac{dt}{t}$$

$$\Rightarrow \int \frac{dy}{y\log y} = \log t$$

$$\Rightarrow \int \frac{dy}{y\log y} = \log(\log y)$$

Evaluating expression (1)

$$\log(\log y) = \log x + \log C$$

$$\Rightarrow \log(\log y) = \log Cx$$

$$\Rightarrow \log y = Cx$$

$$\Rightarrow y = e^{Cx}$$

∴ General solution of given differential equation is $y = e^{Cx}$

8.

Find the general solution for $x^5 \frac{dy}{dx} = -y^5$.

Ans - Given differential equation is

$$x^5 \frac{dy}{dx} = -y^5.$$

$$\frac{dy}{y^5} = -\frac{dx}{x^5}$$

Integrate both side:

$$\int \frac{dy}{y^5} = -\int \frac{dx}{x^{-5}}$$

$$\Rightarrow \int y^{-5} dy = -\int x^{-5} dx$$

$$\Rightarrow \frac{y^{-5+1}}{-5+1} = -\frac{x^{-5+1}}{-5+1} + C$$

$$\Rightarrow \frac{y^{-4}}{-4} = -\frac{x^{-4}}{-4} + C$$

$$\Rightarrow x^{-4} + y^{-4} = -4C$$

$$\Rightarrow x^{-4} + y^{-4} = A \ (A = -4C)$$

∴ General solution of given differential equation is $x^{-4} +$ $y^{-4} = \text{A}$

9.

Find the general solution for $\frac{dy}{dx} = \sin^{-1}x$.

Ans - Given differential equation is

$$\frac{dy}{dx} = \sin^{-1}x$$

$$dy = \sin^{-1}xdx$$

Integrate both side

$$\int dy = \int \sin^{-1}xdx$$

$$\Rightarrow y = \int 1 \times \sin^{-1}xdx$$

Use product rule of integration:

$$\int \sin^{-1}xdx = \sin^{-1}x\int dx - \int \left(\frac{1}{\sqrt{1-x^2}}\int dx\right) dx$$

$$\Rightarrow \int \sin^{-1}xdx = x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx$$

Substitute $1 - x^2 = t^2$

$$1 - x^2 = t^2$$

$$\Rightarrow -2xdx = 2tdt$$

$$\Rightarrow -xdx = tdt$$

Evaluating the integral

$$\frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx + C}{x^2+1}$$

$$\Rightarrow \int \sin^{-1}xdx = x\sin^{-1}x + \int \frac{tdt}{\sqrt{t^2}}$$

$$\Rightarrow \int \sin^{-1}xdx = x\sin^{-1}x + t + C$$

$$\Rightarrow \int \sin^{-1}xdx = x\sin^{-1}x + \sqrt{1-x^2} + C$$

$$\Rightarrow y = x\sin^{-1}x + \sqrt{1-x^2} + C$$

∴ General solution of given differential equation is $y = x\sin^{-1}x + \sqrt{1-x^2} + C$.

10.

Find the general solution for
$\boldsymbol{e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0.}$

Ans - Given differential equation is $e^x \tan y dx + (1 - e^x)\sec^2 y dy = 0.$

$$e^x \tan y dx + (1 - e^x)\sec^2 y dy = 0.$$

Simplify the expression:

$$(1 - e^x)\sec^2 y dy = -e^x \tan y dx$$

$$\Rightarrow \frac{\sec^2 y}{\tan y} dy = -\frac{e^x}{(1 - e^x)} dx$$

Integrate both side:

$$\int \frac{\sec^2 y}{\tan y} dy = -\int \frac{e^x}{(1 - e^x)} dx \quad \ldots\ldots (1)$$

Substitute tan y = u

$$\tan y = u$$

$$\Rightarrow \sec^2 y = du$$

Evaluating the LHS integral of (1):

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy = \int \frac{du}{u}$$

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy = \log u$$

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy = \log(\tan y)$$

Substitute $1 - e^x = v$

$$1 - e^x = \mathbf{v}$$

$$\Rightarrow -e^x dx = dv$$

Evaluating the RHS integral of (1):

$$\Rightarrow -\int \frac{e^x}{(1-e^x)}dx = \int \frac{dv}{v}$$

$$\Rightarrow -\int \frac{e^x}{(1-e^x)}dx = \log v$$

$$\Rightarrow -\int \frac{e^x}{(1-e^x)}dx = \log(1-e^x)$$

Therefore, the integral (1) will be:

$$\log(\tan y) = \log(1-e^x) + \log C$$

$$\Rightarrow \log(\tan y) = \log C(1-e^x)$$

$$\Rightarrow \tan y = C(1-e^x)$$

∴ General solution of given differential equation is $\tan y = C(1-e^x)$.

For each of the differential equations in Exercises 11 to 14, find a particular solution satisfying the given condition:

11.

$$(\mathbf{x^3 + x^2 + x + 1})\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{2x^2 + x;\ y = 1, when\ x = 0}$$

Ans - Given differential equation is

$$(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2+x;\ y=1,\ x=0$$

Simplify the expression

$$\frac{dy}{dx} = \frac{2x^2+x}{(x^3+x^2+x+1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2+x}{(x^3+x+x^2+1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2+x}{(x+1)(x^2+1)}$$

$$\Rightarrow dy = \frac{2x^2+x}{(x+1)(x^2+1)}dx$$

Integrate both side

$$\int dy = \int \frac{2x^2 + x}{(x+1)(x^2+1)} dx \quad \ldots\ldots(1)$$

Use partial fraction method to simplify the RHS:

$$\frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx + C}{x^2+1}$$

$$\Rightarrow 2x^2 + x = A(x^2+1) + (Bx + C)(x+1)$$

$$\Rightarrow 2x^2 + x = (A+B)x^2 + (B+C)x + (A+C)$$

By comparing coefficients:

$\mathrm{A} + \mathrm{B} = 2$
$\mathrm{B} + \mathrm{C} = 1$
$\mathrm{A} + \mathrm{C} = 0$

Solving this we get:

$$\frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{1}{2}\left(\frac{1}{x+1} + \frac{3x-1}{x^2+1}\right)$$

Rewriting the integral (1):

$$y = \frac{1}{2}\int \left(\frac{1}{x+1} + \frac{3x-1}{x^2+1}\right) dx$$

$$\Rightarrow y = \frac{1}{2}\int \frac{1}{x+1} dx + \frac{1}{2}\int \frac{3x-1}{x^2+1} dx$$

$$\Rightarrow y = \frac{1}{2}\log(x+1) + \frac{3}{2}\int \frac{x}{x^2+1} dx - \frac{1}{2}\int \frac{1}{x^2+1} dx$$

$$\Rightarrow y = \frac{1}{2}\log(x+1) + \frac{3}{4}\int \frac{2x}{x^2+1} dx - \frac{1}{2}\tan^{-1}x$$

$$\Rightarrow y = \frac{1}{2}\log(x+1) + \frac{3}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$$

For $y = 1$ when $\mathrm{x} = 0$

$$1 = \frac{1}{2}\log(0+1) + \frac{3}{4}\log(0+1) - \frac{1}{2}\tan^{-1}0 + C$$

$$\Rightarrow C = 1$$

Thus, the required particular solution is:

$$\Rightarrow y = \frac{1}{2}\log(x+1) + \frac{3}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + 1.$$

12.

$\mathbf{x(x^2-1)\frac{dy}{dx}=1;\ y=0}$ **When** $\boldsymbol{x=2}$

Ans - Given differential equation is.

$$x(x^2-1)\frac{dy}{dx}=1; y=0 \text{ when } x=2$$

Simplify the expression

$$x(x^2-1)\frac{dy}{dx}=1$$

$$\Rightarrow dy=\frac{dx}{x(x^2-1)}$$

$$\Rightarrow dy=\frac{dx}{x(x-1)(x+1)}$$

Integrate both side

$$\int dy=\int\frac{dx}{x(x-1)(x+1)}\ \ldots\ldots(1)$$

Use partial fraction method to simplify the RHS:

$$\frac{1}{x(x-1)(x+1)}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}$$

$$\Rightarrow 1=A(x^2-1)+Bx(x+1)+Cx(x-1)$$

$$\Rightarrow 1=(A+B+C)x^2+(B-C)x-A$$

By comparing coefficients:

A+B+C=0

B− C = 0

−A = 1

$$\frac{1}{x(x-1)(x+1)}=\frac{(-1)}{x}+\frac{\left(\frac{1}{2}\right)}{x-1}+\frac{\left(\frac{1}{2}\right)}{x+1}$$

$$\Rightarrow\frac{1}{x(x-1)(x+1)}=-\frac{1}{x}+\frac{1}{2}\left(\frac{1}{x-1}+\frac{1}{x+1}\right)$$

Rewriting the integral (1)

$$y = \int \left(-\frac{1}{x} + \frac{1}{2}\left(\frac{1}{x-1} + \frac{1}{x+1}\right)\right) dx$$

$$\Rightarrow y = -\int \frac{1}{x} dx + \frac{1}{2}\int \frac{1}{x-1} dx + \frac{1}{2}\int \frac{1}{x+1} dx$$

$$\Rightarrow y = -\log x + \frac{1}{2}\log(x-1) + \frac{1}{2}\log(x+1) + \log C$$

$$\Rightarrow y = -\frac{2}{2}\log x + \frac{1}{2}\log(x-1) + \frac{1}{2}\log(x+1) + \frac{2}{2}\log C$$

$$\Rightarrow y = \frac{1}{2}(-\log x^2 + \log(x-1) + \log(x+1) + \log C^2)$$

$$\Rightarrow y = \frac{1}{2}\log\left[\frac{C^2(x^2-1)}{x^2}\right]$$

For $y = 0$ when $x = 2$

$$0 = \frac{1}{2}\log\left[\frac{C^2(2^2-1)}{2^2}\right]$$

$$\Rightarrow 0 = \log\left[\frac{3C^2}{4}\right]$$

$$\Rightarrow \frac{3C^2}{4} = 1$$

$$\Rightarrow C^2 = \frac{4}{3}$$

Thus the required particular solution is

$$y = \frac{1}{2}\log\left[\frac{4(x^2-1)}{3x^2}\right]$$

13.

$\cos\left(\frac{dy}{dx}\right) = \mathbf{a}\ (\mathbf{a} \in \mathbf{R});\ \ \mathbf{y = 1}$ **when** $x = \mathbf{0}$

Ans - Given differential equation is

$$\cos\left(\frac{dy}{dx}\right) = a \ (a \in R); \ y = 1 \text{ when } x = 0$$

Simplify the expression

$$\cos\left(\frac{dy}{dx}\right) = a$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1}a$$

$$\Rightarrow \mathrm{dy} = \cos^{-1}\mathrm{a}\ \mathrm{dx}$$

Integrate both side

$$\int dy = \int \cos^{-1}a dx$$

$$\Rightarrow y = \cos^{-1}a \int dx$$

$$\Rightarrow y = x\cos^{-1}a + C$$

For $y = 1$ when $\mathrm{x} = 0$

$$1 = 0\cos^{-1}a + C$$

$$\Rightarrow \mathrm{C} = 1$$

Thus the required particular is:

$$y = x\cos^{-1}a + 1$$

$$\Rightarrow \frac{y-1}{x} = \cos^{-1}a$$

$$\Rightarrow \cos\left(\frac{y-1}{x}\right) = a$$

14.

$$\frac{\mathbf{dy}}{\mathbf{dx}} = \boldsymbol{y\ tanx}; \mathbf{y} = \mathbf{1} \text{ when } \boldsymbol{x} = \mathbf{0}$$

Ans - Given differential equation is

$$\frac{dy}{dx} = y\ \tan x;\ y = 1 \text{ when } x = 0$$

Simplify the expression:

$$\frac{dy}{dx} = y\tan x$$

$$\Rightarrow \frac{dy}{y} = \tan x dx$$

Integrate both side:

$$\int \frac{dy}{y} = \int \tan x dx$$

$$\Rightarrow \log y = \log(\sec x) + \log C$$

$$\Rightarrow \log y = \log(C\sec x)$$

$$\Rightarrow y = C\sec x$$

For y=1 when x=0.

$$1 = C\sec 0$$

$$\Rightarrow C = 1$$

Thus, the required particular solution is:

$$y = \sec x.$$

15.

Find the equation of a curve passing through the point (0, 0) and whose differential equation is $\mathbf{y' = e^x \sin x}$

Ans - Given differential equation is

$$y' = e^x \sin x$$

The curve passes through (0,0).

Simplify the expression:

$$\Rightarrow \frac{dy}{dx} = e^x \sin x$$

$$\Rightarrow dy = e^x \sin x dx$$

Integrate both side

$\int dy = \int e^x \sin x dx$

Use product rules for integration of RHS. Let

$I = \int e^x \sin x dx$

$\Rightarrow I = \sin x \int e^x dx - \int \left(\cos x \int e^x dx\right) dx$

$\Rightarrow I = e^x \sin x - \int e^x \cos x dx$

$\Rightarrow I = e^x \sin x - \left(\cos x \int e^x dx + \int \left(\sin x \int e^x dx\right) dx\right)$

$\Rightarrow I = e^x \sin x - e^x \cos x - \int (e^x \sin x) dx$

$\Rightarrow I = e^x \sin x - e^x \cos x - I$

$\Rightarrow I = \frac{e^x}{2}(\sin x - \cos x)$

Thus integral will be

$y = \frac{e^x}{2}(\sin x - \cos x) + C$

Thus, as the curve passes through (0,0)

$0 = \frac{e^0}{2}(\sin 0 - \cos 0) + C$

$\Rightarrow 0 = \frac{1}{2}(0 - 1) + C$

$\Rightarrow C = \frac{1}{2}$

Thus, the equation of the curve will be:

$y = \frac{e^x}{2}(\sin x - \cos x) + \frac{1}{2}$

$\Rightarrow y = \frac{e^x}{2}(\sin x - \cos x + 1)$

16.

For the differential equation $xy\frac{dy}{dx} = (x + 2)(y + 2)$ find the solution curve passing through the point (1, -1).

Ans - Given differential equation is

$$xy\frac{dy}{dx} = (x+2)(y+2)$$

The curve passes through (1, -1)

$$\Rightarrow \left(\frac{y}{y+2}\right)dy = \frac{(x+2)}{x}dx$$

$$\Rightarrow \left(1-\frac{2}{y+2}\right)dy = \frac{(x+2)}{x}dx$$

Integrate both side

$$\int \left(1-\frac{2}{y+2}\right)dy = \int \frac{(x+2)}{x}dx$$

$$\Rightarrow \int dy - 2\int \frac{1}{y+2}dy = \int \frac{x}{x}dx + \int \frac{2}{x}dx$$

$$\Rightarrow y - 2\log(y+2) = x + 2\log x + C$$

$$\Rightarrow y - x = 2\log(y+2) + 2\log x + C$$

$$\Rightarrow y - x = 2\log[x(y+2)] + C$$

$$\Rightarrow y - x = \log[x^2(y+2)^2] + C$$

Thus as the curve passes through $(1,-1)$

$$\Rightarrow -1 - 1 = \log[(1)^2(-1+2)^2] + C$$

$$\Rightarrow -2 = \log 1 + C$$

$$\Rightarrow C = -2$$

Thus the equation of the curve will be:

$$y - x = \log[x^2(y+2)^2] - 2$$

$$\Rightarrow y - x + 2 = \log(x^2(y+2)^2)$$

17.

Find the equation of a curve passing through the point (0, -2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y-coordinate of the point is equal to the x coordinate of the point.

Ans - According to the equation is given by

$$y\frac{dy}{dx} = x$$

The curve passes through (0, -2)

Simplify the expression

$$\Rightarrow ydy = xdx$$

Integrate both side

$$\int ydy = \int xdx$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\Rightarrow y^2 - x^2 = 2C$$

Thus, as the curve passes through (0, -2)

$$\Rightarrow (-2)^2 - 0^2 = 2C$$

$$\Rightarrow 4 = 2C$$

$$\Rightarrow C = 2$$

Thus, the equation of the curve will be

$$y^2 - x^2 = 2(2)$$

$$\Rightarrow y^2 - x^2 = 4$$

18.

At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (– 4, –3). Find the equation of the curve given that it passes through (–2, 1).

Ans - Let point of contact of tangent be (x, y). Then the slope of segment joining point of contact and $(-4,-3)$: $m=\frac{y+3}{x+4}$

According to the for the slope of tangent $\frac{\mathrm{dy}}{\mathrm{dx}}$ it follows:

$$\frac{dy}{dx}=2m$$

$$\Rightarrow \frac{dy}{dx}=2\left(\frac{y+3}{x+4}\right)$$

Simplify the expression

$$\frac{dy}{dx}=2\left(\frac{y+3}{x+4}\right)$$

$$\frac{dy}{y+3}=\frac{2}{x+4}dx$$

Integrate both side:

$$\int \frac{dy}{y+3}=\int \frac{2}{x+4}dx$$

$$\Rightarrow \log(y+3)=2\log(x+4)+\log C$$

$$\Rightarrow \log(y+3)=\log(x+4)^2+\log C$$

$$\Rightarrow \log(y+3)=\log C(x+4)^2$$

$$\Rightarrow y+3=C(x+4)^2$$

Thus as the curve passes through $(-2,1)$

$$1+3=C(-2+4)^2$$

$$\Rightarrow 4=4C$$

$$\Rightarrow C=1$$

Thus the equation of the curve will be

$$y+3=(x+4)^2.$$

19.

The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

Ans - Let volume of spherical balloon be V and its radius r.
Let the rate of change of volume be k.

$$\frac{dV}{dt} = k$$

$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = k$$

$$\Rightarrow \frac{4}{3}\pi \frac{d}{dt}(r^3) = k$$

$$\Rightarrow \frac{4}{3}\pi(3r^2)\frac{dr}{dt} = k$$

$$\Rightarrow 4\pi r^2 \frac{dr}{dt} = k$$

$$\Rightarrow 4\pi r^2 dr = kdt$$

Integrate both side:

$$\int 4\pi r^2 dr = \int kdt$$

$$\Rightarrow 4\pi \int r^2 dr = kt + C$$

$$\Rightarrow 4\pi \int r^2 dr = kt + C$$

$$\Rightarrow \frac{4}{3}\pi r^3 = kt + C$$

At initial time, t = 0 and r = 3

$$\frac{4}{3}\pi 3^3 = k(0) + C$$

$$\Rightarrow C = 36\pi$$

$$At\ t = 3\ the\ radius\ r = 6$$

$$\frac{4}{3}\pi(6^3) = k(3) + 36\pi \Rightarrow 3k = 288\pi - 36\pi$$

$$\Rightarrow k = 84\pi$$

Thus, the radius-time relation can be given by

$$\frac{4}{3}\pi r^3 = 84\pi t + 36\pi$$

$$\Rightarrow r^3 = 63t + 27$$

$$\Rightarrow r = (63t + 27)^{\frac{1}{3}}$$

20.

In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs 100 double itself in 10 years ($\log_e 2 = 0.6931$).

Ans - Let the principal be p, according to

$$\frac{dp}{dt} = \left(\frac{r}{100}\right)p$$

Simplify the expression

$$\frac{dp}{p} = \left(\frac{r}{100}\right)dt$$

Integrate both side

$$\int \frac{dp}{p} = \int \left(\frac{r}{100}\right)dt$$

$$\Rightarrow \log p = \frac{rt}{100} + c$$

$$\Rightarrow p = e^{\frac{rt}{100}+c}$$

$$\Rightarrow p = Ae^{\frac{rt}{100}} \quad (A = e^c)$$

$$Att = 0, p = 100$$

$$100 = Ae^{\frac{r(0)}{100}}$$

$$\Rightarrow A = 100$$

Thus, the principle and rate of interest relation will be

$$p = 100e^{\frac{rt}{100}}$$

$$At\ t = 10, p = 2 \times 100 = 200$$

$$200 = 100e^{\frac{r(10)}{100}}$$

$$\Rightarrow 2 = e^{\frac{r}{10}}$$

$$\log\left(e^{\frac{r}{10}}\right) = \log(2)$$

$$\Rightarrow \frac{r}{10} = 0.6931$$

$$\Rightarrow \mathrm{r} = 6.931$$

21.

In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5} = 1.648$).

Ans - Let principal be p then,

$$\frac{dp}{dt} = \left(\frac{5}{100}\right)p$$

$$\frac{dp}{dt} = \frac{p}{20}$$

$$\Rightarrow \frac{dp}{p} = \frac{1}{20}dt$$

Integrate both side

$$\int \frac{dp}{p} = \int \frac{1}{20}dt$$

$$\Rightarrow \log p = \frac{t}{20} + C$$

$$\Rightarrow p = e^{\frac{t}{20}+C}$$

$$\Rightarrow p = Ae^{\frac{t}{20}}(A = e^C)$$

At t = 0, p = 1000

$$1000 = Ae^{\frac{0}{20}}$$

$$\Rightarrow A = 1000$$

Thus, the relation of principal and time relation will be

$$\Rightarrow p = 1000e^{\frac{t}{20}}$$

At t=10: p = 1000e

$$\Rightarrow p = 1000e^{0.5}$$

$$\Rightarrow p = 1000 \times 1.648$$

$$\Rightarrow p = 1648$$

Thus after 10 this year the amount will become Rs.1648

22.

In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

Ans - Let number of bacteria be y at time t.

$$\frac{dy}{dt} \propto y$$

$\frac{dy}{dt} = Cy$, where C is a constant.

Simplify the expression

$$\frac{dy}{y} = cdt$$

Integrate both side:

$$\int \frac{dy}{y} = \int cdt$$

$$\Rightarrow \log y = ct + D$$

$$\Rightarrow y = e^{ct+D}$$

$$\Rightarrow y = Ae^{ct}\ (A = e^{D})$$

At $t = 0$, $y = 100000$

$$100000 = Ae^{c(0)}$$

$$\Rightarrow A = 100000$$

At $t = 2,\ y = \frac{11}{10}(100000) = 110000$

$$y = 100000e^{ct}$$

$$\Rightarrow 110000 = 100000e^{c(2)}$$

$$\Rightarrow e^{2c} = \frac{11}{10}$$

$$\Rightarrow 2c = \log\left(\frac{11}{10}\right)$$

$$\Rightarrow c = \frac{1}{2}\log\left(\frac{11}{10}\right) \ldots\ldots (1)$$

For y = 200000

$200000 = 100000e^{ct}$

$\Rightarrow e^{ct} = 2$

$\Rightarrow ct = \log 2$

$\Rightarrow t = \frac{\log 2}{c}$

Back substituting using expression (1)

$$t = \frac{\log 2}{\frac{1}{a}\log\left(\frac{11}{10}\right)}$$

$$t = \frac{2\log 2}{\log\left(\frac{11}{10}\right)}$$

Thus, time required for bacteria to reach 20000 is

$$t = \frac{\log 2}{\frac{1}{2}\log\left(\frac{11}{10}\right)} \text{ hrs}$$

23.

Find the general solution of the differential equation $\frac{dy}{dx} =$ e^{x+y}

(A) $e^{x} + e^{-y} = C$ **(B)** $e^{x} + e^{y} = C$

(C) $e^{-x} + e^{y} = C$ **(D)** $e^{-x} + e^{-y} = C$

Ans - Given differential equation is $\frac{dy}{dx} = e^{x+y}$

Simplify the expression:

$$\frac{dy}{dx} = e^x e^y$$

$$\Rightarrow \frac{dy}{e^y} = e^x dx$$

$$\Rightarrow e^{-y} dy = e^x dx$$

Integrate both side:

$$\int e^{-y} dy = \int e^x dx$$

$$\Rightarrow -e^{-y} = e^x + D$$

$$\Rightarrow e^x + e^{-y} = -D$$

$$\Rightarrow e^x + e^{-y} = C \ (C = -D)$$

∴ General solution of given differential equation is $e^x + e^{-y} = C$

Hence the correct option is (A).

Exercise 9.4

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve each of them.

1.

$$(x^2 + xy)dy = (x^2 + y^2)dy$$

Ans - After rearranging the given equation we get

$$\frac{dy}{dx}=\frac{x^2+y^2}{x^2+xy}$$

It is a homogeneous differential equation.

To solve this problem, we will make the substitution,
$y=vx$

Differentiating equation w.r.t. x we get

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substituting $y=vx$ and $\frac{dy}{dx}$ in the above question, we get

$$v+x\frac{dv}{dx}=\frac{x^2+(vx)^2}{x^2+x(vx)}$$

$$v+x\frac{dv}{dx}=\frac{x^2}{x^2}\left(\frac{1+v^2}{1+v}\right)$$

$$v+x\frac{dv}{dx}=\frac{1+v^2}{1+v}$$

$$x\frac{dv}{dx}=\frac{1+v^2}{1+v}-v$$

$$x\frac{dv}{dx}=\frac{1+v^2-v-v^2}{1+v}$$

$$x\frac{dv}{dx}=\frac{1-v}{1+v}$$

$$\frac{1}{x}dx=\frac{1+v}{1-v}dv$$

2.

$$\boldsymbol{y'=\frac{x+y}{x}}$$

Ans - $\frac{dy}{dx} = \frac{x+y}{x}$

On rearranging the equation,we ge

$$\frac{dy}{dx} = 1 + \frac{y}{x}$$

It is a homogeneous differential equation.

To solve this problem, we will make the substitution.

$$y = vx$$

Differentiating equation w.r.t. x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substiting $y = vx$ and $\frac{dy}{dx}$ in the above equation we get

$$v + x\frac{dv}{dx} = 1 + v$$

$$dv = \frac{1}{x}dx$$

Taking integration on both side

$$\int dv = \int \frac{1}{x}dx$$

$$v = logx + C$$

Substutigthe value of $v = \frac{y}{x}$

$$\frac{y}{x} = \log x + C$$

$$y = x\log x + Cx$$

This is the required differential equation.

3.

$$(\boldsymbol{x - y})\boldsymbol{dy} = (\boldsymbol{x + y})\boldsymbol{dx}$$

Ans - After rearranging the given equation we get

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

It is a homogeneous differential equation.

To solve this problem. We will make the substitution.
$y = vx$

Differentiating equation wrt. x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting $y = vx$ and $\frac{dy}{dx}$ in the above equation, we get

$$v + x\frac{dv}{dx} = \frac{x+vx}{x-vx}$$

$$v + x\frac{dv}{dx} = \frac{1+v}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+\mathrm{v}}{1-\mathrm{v}} - \mathrm{v}$$

$$x\frac{dv}{dx} = \frac{1+v-v+v^2}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{1-v}$$

$$\frac{1-v}{1+v^2}dv = \frac{1}{x}dx$$

Taking integration on both side,

$$\int \frac{1-v}{1+v^2}dv = \int \frac{1}{\mathrm{x}}dx$$

$$\int \frac{1}{1+v^2}dv - \int \frac{v}{1+v^2}dv = \int \frac{1}{\mathrm{x}}dx \qquad \ldots\ldots(1)$$

Let $I = \int \frac{v}{1+v^2}dv$

Now, let $1 + v^2 = t$

Differentiating equation w.r.t. v we get

$$2udu = dt$$

$$vdv = \frac{dt}{2}$$

Substituting $vdv = \frac{dt}{2}$ in the above equation, we get

$$I = \int \frac{1}{2t} dt$$

Substituting this value in equation (1)

$$\Rightarrow \int \frac{1}{1+v^2} dv - \int \frac{1}{2t} dv = \int \frac{1}{\text{x}} dx$$

$$\tan^{-1} v - \frac{1}{2} \log t = \log x + C$$

$$\int \frac{1}{t} dt = -\int \frac{1}{x} dx$$

$$\log t = -\log x + C$$

Substituting the value of $1 + v^2 = t$ and $v = \frac{y}{x}$

$$\log\left(1 + \frac{y^2}{x^2}\right) = -\log x + C$$

$$\log\left(1 + \frac{y^2}{x^2}\right) + \log x = C$$

$$\log\left(\frac{x^2 + y^2}{x^2} \times x\right) = C$$

$$\frac{x^2 + y^2}{x} = e^C$$

$$\frac{x^2 + y^2}{x} = K$$

$$x^2 + y^2 = Kx$$

This is the required differential equation.

4.

$$(\boldsymbol{x^2 - y^2})\boldsymbol{dx + 2xydy = 0}$$

Ans - After rearranging the given equation we get

$$\frac{dy}{dx} = -\frac{x^2 - y^2}{2xy}$$

It is a homogeneous differential equation.

To solve this problem. we will make the substitution,
$y = \mathrm{v}x$

Differentiating equation w.r.t. x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting $y = vx$ and $\frac{dy}{dx}$ in the above equation, we get

$$v + x\frac{dv}{dx} = -\frac{x^2 - (vx)^2}{2x.\mathrm{v}x}$$

$$v + x\frac{dv}{dx} = -\frac{1 - v^2}{2v}$$

$$x\frac{dv}{dx} = -\frac{1 - v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{-1 + v^2 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$$

$$-\frac{2v}{1 + v^2}dv = \frac{1}{x}dx$$

$$\frac{2v}{1 + v^2}dv = -\frac{1}{x}dx$$

Taking integration on both sides,

$$\int \frac{2v}{1 + v^2}dv = -\int \frac{1}{x}dx \qquad \dots\dots(1)$$

Now, let $1 + v^2 = t$

Differentiating equation w.r.t. v, we get

$$2vdv = dt$$

Substituting $2vdv = dt$ in equation (1), we get

$$\int \frac{1}{t} dt = -\int \frac{1}{x} dx$$

$$\log t = -\log x + C$$

Substituting the value of $1 + v^2 = t$ and $v = \frac{y}{x}$

$$\log\left(1 + \frac{y^2}{x^2}\right) = -\log x + C$$

$$\log\left(1 + \frac{y^2}{x^2}\right) + \log x = C$$

$$\log\left(\frac{x^2 + y^2}{x^2} \times x\right) = C$$

$$\frac{x^2 + y^2}{x} = e^C$$

$$\frac{x^2 + y^2}{x} = K$$

$$x^2 + y^2 = Kx$$

This is the required differential equation.

5.

$$\boldsymbol{x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy}$$

Ans - After rearranging the given equation we get

$$\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2}$$

It is a homogeneous differential equation.

To solve this problem, we will make the substitution.,
$y = vx$

Differentiating equation w.r.t. x we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting $\frac{dy}{dx}$ in the above equation, we get

$$v + x\frac{dv}{dx} = 1 - 2v^2 + v$$

$$x\frac{dv}{dx} = 1 - 2v^2$$

$$\frac{1}{1-2v^2}dv = \frac{1}{x}dx$$

Taking integration on both sides

$$\int \frac{1}{1-2v^2}dv = \int \frac{1}{x}dx$$

$$\int \frac{1}{1^2 - \left(\sqrt{2}v\right)^2}dv = \int \frac{1}{x}dx$$

On integrating using standard trigonometric identity we get,

On integrating using standard trigonometric identity we get,

$$\frac{1}{\sqrt{2}}.\frac{1}{1.2}.\log\left|\frac{1+\sqrt{2}v}{1-\sqrt{2}v}\right| = \log|x| + C$$

Substituting the value of $v = \frac{y}{x}$.

$$\frac{1}{2\sqrt{2}}\log\left|\frac{1+\sqrt{2}\frac{y}{x}}{1-\sqrt{2}\frac{y}{x}}\right| = \log|x| + C$$

$$\frac{1}{2\sqrt{2}}\log\left|\frac{x+\sqrt{2}y}{x-\sqrt{2}y}\right| = \log|x| + C$$

This is the required differential equation.

6.

$$\boldsymbol{xdy - ydx = \sqrt{x^2 + y^2}dx}$$

Ans - After rearranging the given equation we get,

$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

$$\frac{dy}{dx} = \sqrt{1 + \frac{y^2}{x^2}} + \frac{y}{x}$$

It is a homogeneous differential equation

To solve this problem, we will make the substitution

$y = vx$

Differentiating equation w.r.t. x we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting $y = vx$ and $\frac{dy}{dx}$ in the above, equation, we get

$$v + x\frac{dv}{dx} = \sqrt{1 + v^2} + v$$

$$x\frac{dv}{dx} = \sqrt{1 + v^2}$$

$$\frac{1}{\sqrt{1 + v^2}}dv = \frac{1}{x}dx$$

Taking integration on both sides,

$$\int \frac{1}{\sqrt{1 + v^2}}dv = \int \frac{1}{x}dx$$

Using $\int \frac{1}{\sqrt{a^2+x^2}} = \log\left|x + \sqrt{a^2 + x^2}\right| + C, we\ get$

$$\log\left|v + \sqrt{1 + v^2}\right| = \log x + \log C$$

Substituting the value of $v = \frac{y}{x}$

$$\log\left|\frac{y}{x} + \frac{1}{\sqrt{1 + \left(\frac{y}{x}\right)^2}}\right| = \log xC$$

$$\frac{y}{x} + \frac{1}{\sqrt{1 + \left(\frac{y}{x}\right)^2}} = xC$$

$$\frac{y}{x} + \sqrt{\frac{x^2 + y^2}{x^2}}$$

$$\frac{y}{x} + \sqrt{\frac{x^2 + y^2}{x}} = xC$$

$$y = \sqrt{x^2 + y^2} = Cx^2$$

This is the required differential equation.

7.

$$\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\} ydx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\} x$$

Ans - After rearranging the given equation we get,

$$\frac{dy}{dx} = \frac{\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\} y}{\left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\} x}$$

It is a homogeneous differential equation

To solve this problem, we will make substitution, $y = vx$

Differentiating equation w.r.t. , we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting $y = vx$ and $\frac{dy}{dx}$ in the above equation we get,

$$v + x\frac{dv}{dx} = \frac{\{x\cos(v) + vx\sin(v)\}vx}{\{vx\sin(v) - x\cos(v)\}x}$$

$$x\frac{dv}{dx} = \frac{\{\cos(v) + v\sin(v)\}v}{\{v\sin(v) - \cos(v)\}} - v$$

$$x\frac{dv}{dx} = \frac{v\cos(v) + v^2\sin(v) - v^2\sin(v) + v\cos(v)}{v\sin(v) - \cos(v)}$$

$$x\frac{dv}{dx} = \frac{2v\cos(v)}{v\sin(v) - \cos(v)}$$

$$\frac{v\sin(v) - \cos(v)}{2v\cos(v)}dv = \frac{1}{x}dx$$

Taking integration on both sides

$$\int \frac{v\sin(v) - \cos(v)}{2v\cos(v)}dv = \int \frac{1}{x}dx$$

$$\int \frac{v\sin(v)}{2v\cos(v)}dv - \int \frac{\cos(v)}{2v\cos(v)}dv = \int \frac{1}{x}dx$$

$$\frac{1}{2}\int \tan v dv - \frac{1}{2}\int \frac{1}{v}dv = \int \frac{1}{x}dx$$

$$\frac{1}{2}\log\sec v - \frac{1}{2}\log v = \log x + \log C$$

$$\frac{1}{2}(\log\sec v - \log v) = \log xC$$

$$\log\frac{\sec v}{v} = 2\log xC$$

$$\log\frac{\sec v}{v} = \log(xC)^2$$

$$\frac{\sec v}{v} = (xC)^2$$

Substituting the value of $v = \frac{y}{x}$

$$\frac{\sec\left(\frac{y}{x}\right)}{\frac{y}{x}} = (xC)^2$$

$$\frac{y}{x}\sec\left(\frac{y}{x}\right) = (xC)^2$$

$$\frac{x}{yx^2}.\frac{1}{\cos\left(\frac{y}{x}\right)} = C^2$$

$$\frac{1}{yx} = \frac{1}{\cos\left(\frac{y}{x}\right)} = C^2$$

$$\frac{1}{C^2} = yx\cos\left(\frac{y}{x}\right)$$

$$yx\cos\left(\frac{y}{x}\right) = K$$

This is the required differential equation.

8.

$$\boldsymbol{x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0}$$

Ans - After rearranging the given equation we get,

$$\frac{dy}{dx} = \frac{y - x\sin\left(\frac{y}{x}\right)}{x}$$

$$\frac{dy}{dx} = \frac{y}{x} - \sin\left(\frac{y}{x}\right)$$

It is a homogeneous differential equation

To solve this problem, we will make substitution, $y = vx$

Differentiating equation w.r.t. x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting $y = vx\ and\ \frac{dy}{dx}$ in the above equation, we get

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \sin\left(\frac{vx}{x}\right)$$

$$v + x\frac{dv}{dx} = v - \sin v$$

$$x\frac{dv}{dx} = -\sin v$$

$$\frac{1}{\sin v}dv = -\frac{1}{x}dx$$

Taking integration on both sides,

$$\int \frac{1}{\sin v}dv = -\int \frac{1}{x}dx$$

$$\int \cos ecv\ dv = -\int \frac{1}{x}dx$$

$$\log(\operatorname{cosec} v - \cot v) = -\log x + \log C$$

$$\log(\operatorname{cosec} v - \cot v) = \log\frac{C}{x}$$

$$\operatorname{cosec} v - \cot v = \frac{C}{x}$$

Substituting the value of $v = \frac{y}{x}$

$$\text{cosec}\left(\frac{y}{x}\right) - \cot\left(\frac{y}{x}\right) = \frac{C}{x}$$

$$\frac{1}{\sin\left(\frac{y}{x}\right)} - \frac{\cos\left(\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{C}{x}$$

$$1 - \cos\left(\frac{y}{x}\right) = \frac{C}{x}\sin\left(\frac{y}{x}\right)$$

$$x\left(1 - \cos\left(\frac{y}{x}\right)\right) = C\sin\left(\frac{y}{x}\right)$$

This is the required differential equation.

9.

$$\boldsymbol{ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0}$$

Ans - After rearranging the given equation we get,

$$\frac{dy}{dx} = \frac{-y}{x\log\left(\frac{y}{x}\right) - 2x}$$

$$\frac{dy}{dx} = \frac{y}{2x - x\log\left(\frac{y}{x}\right)}$$

It is a homogeneous differential equation

To solve this problem, we will make substitution, $y = vx$

Differentiating equation w.r.t. x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting $y = vx$ and $\frac{dy}{dx}$ in the above equation, we get

$$v + x\frac{dv}{dx} = \frac{vx}{2x - x\log\left(\frac{y}{x}\right)}$$

$$v + x\frac{dv}{dx} = \frac{v}{2 - \log v}$$

$$x\frac{dv}{dx} = \frac{v}{2 - \log v} - v$$

$$x\frac{dv}{dx} = \frac{v - 2v + v\log v}{2 - \log v}$$

$$x\frac{dv}{dx} = \frac{-v + v\log v}{2 - \log v}$$

$$\frac{2 - \log v}{v(\log v - 1)}dv = \frac{1}{x}dx$$

$$\frac{1 - (\log v - 1)}{v(\log v - 1)}dv = \frac{1}{x}dx$$

$$\frac{1}{v(\log v - 1)}dv - \frac{1}{v}dv = \frac{1}{x}dx \qquad \ldots\ldots(1)$$

Taking integration on both sides,

$$\int \frac{1}{v(\log v - 1)}dv - \int \frac{1}{v}dv = \int \frac{1}{x}dx \qquad \ldots.(1)$$

Let $I = \int \frac{1}{v(\log v - 1)}dv$

Put $\log v - 1 = t$

Differentiating w.r.t v

$$\frac{1}{v} = \frac{dt}{dv}$$

$$\frac{1}{v}dv = dt$$

$$\Rightarrow I = \int \frac{1}{t}dt$$

Substitute this in equation (1)

$$\int \frac{1}{t}dt - \int \frac{1}{v}dt = \int \frac{1}{x}dt$$

Substituting the value of $\log v - 1$ $and\ v = \frac{y}{x}$

$$\log(\log v - 1) = \log x + \log c$$

$$\log\left(\frac{\log\left(\frac{y}{x}\right) - 1}{\left(\frac{y}{x}\right)}\right) = \log xC$$

$$\frac{\log\left(\frac{y}{x}\right) - 1}{\left(\frac{y}{x}\right)} = xC$$

$$\log\left(\frac{y}{x}\right) - 1 = \frac{y}{x}.xC$$

$$\log\left(\frac{y}{x}\right) - 1 = yC$$

This is the required differential equation.

10.

$$\left(\mathbf{1} + \boldsymbol{e^{\frac{x}{y}}}\right)\boldsymbol{dx} + \boldsymbol{e^{\frac{x}{y}}}\left(\mathbf{1} - \frac{\boldsymbol{x}}{\boldsymbol{y}}\right)\boldsymbol{dy} = \mathbf{0}$$

Ans - After rearranging the given equation we get,

$$\frac{dx}{dy} = \frac{-e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)}{\frac{x}{1 + e^{\frac{x}{y}}}}$$

It is a homogeneous differential equation.

To solve this problem, we will make substitution, $x = vy$

Differentiating equation w.r.t. y, we get

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

Substituting $x = vy\ and\ \frac{dy}{dx}$ in the above equation, we get

$$v+y\frac{dv}{dy}=\frac{-e^{\frac{vy}{y}}\left(1-\frac{vy}{y}\right)}{\frac{vy}{y}}$$

$$v+y\frac{dv}{v}=\frac{-e^{v}(1-v)}{(1+e^{v})}$$

$$v+y\frac{dv}{dy}=\frac{-e^{v}+ve^{v}}{1+e^{v}}$$

$$y\frac{dv}{dy}=\frac{-e^{v}+ve^{v}}{1+e^{v}}-v$$

$$y\frac{dv}{dy}=\frac{-e^{v}+ve^{v}-v-ve^{v}}{1+e^{v}}$$

$$y\frac{dv}{dy}=\frac{-e^{v}-v}{1+e^{v}}$$

$$y\frac{dv}{dy}=-\left[\frac{e^{v}+v}{1+e^{v}}\right]$$

$$\left[\frac{1+e^{v}}{e^{v}+v}\right]dv=-\frac{1}{y}dy$$

Taking integration on both sides,

$$\int\left[\frac{1+e^{v}}{e^{v}+v}\right]dv=-\int\frac{1}{y}dy$$

On integrating both sides,

$$\log(v+e^{v})=-\log y+\log C$$

Substituting the value of $v=\frac{x}{y}$.

$$\log\left(\frac{x}{y}+e^{\frac{x}{y}}\right)=-\log y+\log C$$

$$\log\left(\frac{x}{y}+e^{\frac{x}{y}}\right)=\log\left(\frac{C}{y}\right)$$

$$\frac{x}{y}+e^{\frac{x}{y}}=\frac{C}{y}$$

$$x+ye^{\frac{x}{y}}=C$$

This is the required differential equation.

For each of the differential equations in Exercises from 11 to 15, find the particular solution satisfying the given condition:

11.

$(x+y)dy+(x-y)dx=0; y=1$ **when** $x=1$

Ans - After rearranging the given equation we get

$$\frac{dy}{dx}=-\frac{(x-y)}{(x+y)}$$

It is a homogeneous differential equation.

To solve this problem, we will make substitution, $y=vx$

Differentiating equation w.r.t. x we get

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substituting $y=vx$ and $\frac{dy}{dx}$ in the above equation, we get

$$v+x\frac{dv}{dx}=-\frac{(x-vx)}{(x+vx)}$$

$$v+x\frac{dv}{dx}=-\frac{(1-v)}{(1+v)}$$

$$x\frac{dv}{dx}=-\frac{(1-v)}{(1+v)}-v$$

$$x\frac{dv}{dx}=\frac{-1+v-v-v^2}{1+v}$$

$$x\frac{dv}{dx}=\frac{-1-v^2}{1+v}$$

$$\frac{1+v}{1+v^2}dv=-\frac{1}{x}dx$$

Taking integration on both sides,

$$\int \frac{1+v}{1+v^2} dv = -\int \frac{1}{x} dx$$

$$\int \frac{1}{1+v^2} dv + \int \frac{v}{1+v^2} dv = -\int \frac{1}{x} dx$$

$$\tan^{-1} v + \frac{1}{2} \log(1+v^2) = -\log x + C$$

Substituting the value of $v = \frac{y}{x}$

$$\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)^2\right) = -\log x + C$$

$y = 1$

When $x = 1$

$$\tan^{-1}\left(\frac{1}{1}\right) + \frac{1}{2}\log\left(1+\left(\frac{1}{1}\right)^2\right) = -\log 1 + C$$

$$\frac{\pi}{4} + \frac{1}{2}\log(2) = C$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) + \frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)^2\right) = -\log x + \frac{\pi}{4} + \frac{1}{2}\log(2)$$

$$2\tan^{-1}\left(\frac{y}{x}\right) + \log\left(\frac{x^2+y^2}{x^2}\right)$$
$$= -2\log x + 2 \times \frac{\pi}{4} + 2 \times \frac{1}{2}\log(2)$$

$$2\tan^{-1}\left(\frac{y}{x}\right) + \log\left(\frac{x^2+y^2}{x^2}\right) = -\log x^2 + \frac{\pi}{2} + \log(2)$$

$$2\tan^{-1}\left(\frac{y}{x}\right) + \log\left(\frac{x^2+y^2}{x^2}\right) + \log x^2 = +\frac{\pi}{2} + \log(2)$$

$$2\tan^{-1}\left(\frac{y}{x}\right) + \log\left(\frac{x^2+y^2}{x^2} \times x^2\right) = +\frac{\pi}{2} + \log(2)$$

$$2\tan^{-1}\left(\frac{y}{x}\right) + \log(x^2+y^2) = \frac{\pi}{2} + \log(2)$$

This is the required differential equation

12.

$\boldsymbol{x^2dy + (xy + y^2)dx = 0; y = 1}$ **when** $\boldsymbol{x = 1}$

Ans - After rearranging the given equation we get

$$\frac{dy}{dx} = -\frac{xy + y^2}{x^2}$$

It is a homogeneous differential equation

To solve this problem. we will make substitution, $y = vx$

Differentiating equation w.r.t. x we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting $y = vx$ and $\frac{dy}{dx}$ in the above equation, we get

$$v + x\frac{dv}{dx} = -\frac{xvx + (xv)^2}{x^2}$$

$$v + x\frac{dv}{dx} = -\frac{vx^2 + x^2v^2}{x^2}$$

$$v + x\frac{dv}{dx} = -v - v^2$$

$$x\frac{dv}{dx} = -v - v^2 - v$$

$$x\frac{dv}{dx} = -2v - v^2$$

$$\frac{1}{2v + v^2}dv = -\frac{1}{x}dx$$

Taking integration on both sides,

$$\int \frac{1}{2v + v^2}dv = -\int \frac{1}{x}dx$$

$$\int \frac{1}{v(2 + v)}dv = --\int \frac{1}{x}dx$$

Dividing and multiplying above equation by (2)

$$\frac{1}{2}\int \frac{2}{v(2+v)}dv = -\int \frac{1}{x}dx$$

$$\frac{1}{2}\int \frac{2+v-v}{v(2+v)}dv = -\int \frac{1}{x}dx$$

$$\frac{1}{2}\int \frac{2+v}{v(2+v)}dv - \frac{1}{2}\int \frac{v}{v(2+v)}dv = -\int \frac{1}{x}dx$$

$$\frac{1}{2}\int \frac{1}{v}dv - \frac{1}{2}\int \frac{1}{(2+v)}dv = -\int \frac{1}{x}dx$$

$$\frac{1}{2}\log v - \frac{1}{2}\log(2+v) = -\log x + \log C$$

$$\frac{1}{2}\log\left(\frac{v}{2+v}\right) = \log\frac{C}{x}$$

$$\log\left(\frac{v}{2+v}\right) = 2\log\frac{C}{x}$$

$$\mathrm{og}\left(\frac{v}{2+v}\right) = \log\left(\frac{C}{x}\right)^2$$

$$\frac{v}{2+v} = \left(\frac{C}{x}\right)^2$$

Substituting the value of $v = \frac{y}{x}$

$$\frac{\frac{y}{x}}{2+\frac{y}{x}} = \left(\frac{C}{x}\right)^2$$

$$\frac{\frac{y}{x}}{\frac{2x+y}{x}} = \left(\frac{C}{x}\right)^2$$

$$\frac{y}{2x+y} = \left(\frac{C}{x}\right)^2$$

$y = 1$

When $x = 1$

$$\frac{y}{2.1+y} = \left(\frac{C}{1}\right)^2$$

$$\frac{1}{3} = C^2$$

⇒ Final solution becomes,

$$\frac{y}{2x+y} = \left(\frac{C}{x}\right)^2$$

$$\frac{y}{2x+y} = \frac{\mathrm{C}^2}{\mathrm{x}^2}$$

$$\frac{y\mathrm{x}^2}{2x+y} = \mathrm{C}^2$$

$$\frac{y\mathrm{x}^2}{2x+y} = \frac{1}{3}$$

$2x + y = 3y\mathrm{x}^2$

This is the required differential equation.

13.

$$\left[x \sin^2\left(\frac{y}{x}\right) - y\right] dx + x dy = 0;\ y = \frac{\pi}{4} \text{ when } x = 1.$$

Ans - After rearranging the given equation we get

$$\frac{dy}{dx} = \frac{\left[x\sin^2\left(\frac{y}{x}\right) - y\right]}{x}$$

It is a homogeneous differential equation

To solve this problem. we will make substitution, $y = vx$

Differentiating equation w.r.t. x we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting $y = vx$ and $\frac{dy}{dx}$ in the above equation, we get

$$v + x\frac{dv}{dx} = -\frac{\left[x\sin^2\left(\frac{vx}{x}\right) - vx\right]}{x}$$

$$v + x\frac{dv}{dx} = -\sin^2(v) + v$$

$$x\frac{dv}{dx} = -\sin^2(v) + v - v$$

$$x\frac{dv}{dx} = -\sin^2(v)$$

$$\frac{1}{\sin^2(v)}dv = -\frac{1}{x}dx$$

$$\text{cosec } vdv = -\frac{1}{x}dx$$

Taking integration on both sides,

$$\int cosec\ vdv = -\int \frac{1}{x}dx$$

$$-\cot v = -\log x - \log C$$

$$\cot v = \log x + \log C$$

$$\cot v = \log xC$$

Substituting the value of $v = \frac{y}{x}$

$$\cot\left(\frac{y}{x}\right) = \log xC$$

$$y = \frac{\pi}{4}$$

When $x = 1$

$$\cot\left(\frac{\frac{\pi}{4}}{1}\right) = \log 1C$$

$$\cot\left(\frac{\pi}{4}\right) = \log C$$

$$1 = \log C$$

$$e^1 = C$$

$$e = C$$

⇒ Final solution becomes

$$\cot\left(\frac{y}{x}\right) = \log|xe|$$

This is the required differential equation.

14.

$$\boldsymbol{\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0, y = 0 \text{ when } x = 1}$$

Ans - After rearranging the given equation we get

$$\frac{dy}{dx}=\frac{y}{x}-\operatorname{cosec}\left(\frac{y}{x}\right)$$

It is a homogeneous differential equation

To solve this problem. we will make substitution, $y=vx$

Differentiating equation w.r.t. x.we get.

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substituting $y=vx$ and $\frac{dy}{dx}$ in the above quation, we get

$$v+x\frac{dv}{dx}=\frac{vx}{x}-cosec\left(\frac{vx}{x}\right)$$

$$v+x\frac{dv}{dx}=v-\operatorname{cosec}(v)$$

$$x\frac{dv}{dx}=-\operatorname{cosec}(v)$$

$$\frac{1}{\operatorname{cosec}(v)}dv=-\frac{1}{x}dx$$

$$\sin v\,dv=-\frac{1}{x}dx$$

Taking integration on both sides

$$\int\sin v\,dv=-\int\frac{1}{x}dx$$

$$-\cos v=-\log x+C$$

Substituting the value of $v=\frac{y}{x}$

$$-\cos\left(\frac{y}{x}\right)=-\log x+C$$

$$y=0$$

When $\mathrm{x}=1$

$$-\cos\left(\frac{0}{1}\right)=-\log 1+C$$

$$-1=C$$

⇒ Final solution becomes,

$$-\cos\left(\frac{y}{x}\right) = -\log x - 1$$

$$\cos\left(\frac{y}{x}\right) = \log x + 1$$

$$\cos\left(\frac{y}{x}\right) = \log x + \log e$$

$$\cos\left(\frac{y}{x}\right) = \log|xe|$$

This is the required differential equation.

15.

$$\boldsymbol{2xy + y^2 - 2x^2\frac{dy}{dx} = 0, y = 2 \text{ When } x = 1}$$

Ans - After rearranging the given equation we get

$$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$$

It is a homogeneous differential eguation

To solve this problem.we will make substitution, $y = vx$

Differentiating equation w.r.t. x we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting $y = vx$ and $\frac{dy}{dx}$ in above equation

$$v + x\frac{dv}{dx} = \frac{2xvx + (vx)^2}{2x^2}$$

$$v + x\frac{dv}{dx} = \frac{2x^2v + v^2x^2}{2x^2}$$

$$v + x\frac{dv}{dx} = \frac{2v + v^2}{2}$$

$$x\frac{dv}{dx} = \frac{2v + v^2}{2} - v$$

$$x\frac{dv}{dx}=\frac{2v+v^2-2v}{2}$$

$$x\frac{dv}{dx}=\frac{v^2}{2}$$

$$\frac{2}{v^2}dv=\frac{1}{x}dx$$

Taking integration on both sides,

$$\int\frac{2}{v^2}dv=\int\frac{1}{x}dx$$

Substituting the value of $v=\frac{y}{x}$

$$-\frac{2}{y}=\log x+C$$

$$x$$

$$-\frac{2x}{y}=\log x+C$$

$$y=2$$

when $x=1$

$$-\frac{2.1}{2}=\log 1+C$$

$$-1=C$$

Therefore, final solution becomes,

$$-\frac{2x}{y}=\log x-1$$

$$\frac{2x}{y}=1-\log x$$

$$y=\frac{2x}{1-\log x}:x\neq e$$

This is the required differential equation.

16.

A homogeneous differential equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution.

(A) $\mathbf{y = vx}$ (B) $\mathbf{v = yx}$

(C) $\mathbf{x = vy}$ (D) $\mathbf{x = v}$

Ans - As $h\left(\frac{x}{y}\right)$ is function of $\frac{x}{y}$

Therefore, we have to substitute, $x = vy$

Hence, the correct answer is option C.

17.

Which of the following is a homogeneous differential equation?

(A) $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$

(B) $(xy)dx - (x^3 + y^3)dy = 0$

(C) $(x^3 + 2y^2)dx + 2xydy = 0$

(D) $y^2dx + (x^2 - xy - y^2)dy = 0$

Ans - Correct answer is option D.

Explanation

$y^2dx + (x^2 - xy - y^2)dy = 0$

After rearranging the given equation we get.

$$\frac{dx}{dy} = -\frac{x^2 - xy - y^2}{y^2}$$

Let $f(x, y) = -\frac{x^2 - xy - y^2}{y^2}$

Now, put $x = kx \; and \; y = ky$

$$f(kx, ky) = -\frac{(kx)^2 - kxky - (ky)^2}{(ky)^2}$$

$$f(kx, ky) = -\frac{k^2}{k^2}\frac{x^2 - xy - y^2}{y^2}$$

$$f(kx, ky) = k^0\left(-\frac{x^2 - xy - y^2}{y^2}\right)$$

$$f(kx, ky) = k^0 f(x, y)$$

Hence, the given differential equation is homogenous.

Exercise 9.5

For each of the differential equations given in Exercises 1 to 12, find the general solution:

1.

$$\frac{dy}{dx} + 2y = sinx$$

Ans - Given equation is in the form of $\frac{dy}{dx} + Py = Q$

(Where, $P = 2$ and $Q = sin\ x$

Calculating integration factor,

$I.F. = e^{\int Pdx}$

$I.F. = e^{\int 2dx}$

$I.F. = e^{2x}$

$\therefore$ Solution of given differential equation is given by relation,

$y(I.F.) = \int (Q \times I.F.)dx + C$

$ye^{2x} = \int \sin x. e^{2x} dx + C \ldots\ldots (1)$

Let $I = \int \sin x. e^{2x} dx$

Using integration by parts,

$$I = \sin x \int e^{2x} dx - \int \left(\frac{d(\sin x)}{dx}. \int e^{2x} dx\right) dx$$

$$I = \frac{e^{2x}\sin x}{2} - \int \cos x. \frac{e^{2x}}{2} dx$$

Again using integration by parts,

$$I = \frac{e^{2x}\sin x}{2} - \cos x \int \frac{e^{2x}}{2} dx + \int \left(\frac{d(\cos x)}{dx}. \int \frac{e^{2x}}{2} dx\right) dx$$

$$I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} + \int (-\sin x). \frac{e^{2x}}{4} dx$$

$$I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}\int \sin x. e^{2x} dx$$

As $I = \int \sin x . e^{2x} dx$,

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}I$$

$$I + \frac{1}{4}I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}$$

$$\frac{5}{4}I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}$$

$$I = \frac{e^{2x}}{5}(2\sin x - \cos x)$$

Substituting this value of I in equation (1).

$$ye^{2x} = \frac{e^{2x}}{5}(2\sin x - \cos x) + C$$

$$y = \frac{1}{5}(2\sin x - \cos x) + Ce^{-2x}$$

2.

$$\boldsymbol{\frac{dy}{dx} + 3y = e^{-2x}}$$

Ans - Given equation is in the form of $\frac{dy}{dx} + Py = Q$

Where, $P = 3$ and $Q = e^{-2x}$

Calculating integration factor,

$I.F = e^{\int Pdx}$

$I.F = e^{\int 3dx}$

$I.F = e^{3x}$

$\therefore$ Solution of given differential equation is given by relation,

$y(I.F.) = \int (Q \times I.F.)dx + C$

$ye^{3x} = \int (e^{3x} \times e^{-2x})dx + C$

$ye^{3x} = \int e^{x}dx + C$

$ye^{3x} = e^{x} + C$

$y = e^{-2x} + Ce^{x}$

3.

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

Ans - Given equation is in the form of $\frac{dy}{dx} + Py = Q$

Where, $P = \frac{1}{x} and\ Q = x^2$

Calculating integration factor,

$I.F = e^{\int Pdx}$

$I.F = e^{\int \frac{1}{x}dx}$

$I.F = e^{\log x}$

$I.F = x$

∴ Solution of given differential equation is given by relation,

$$y(I.F.) = \int (Q \times I.F.)dx + C$$

$$yx = \int (x^2.x)dx + C$$

$$yx = \int (x^3)dx + C$$

$$yx = \frac{x^4}{4} + C$$

4.

$$\frac{dy}{dx} + (\sec x)y = \tan x \left(0 \le x \le \frac{\pi}{2}\right)$$

Ans - Given equation is in the form of $\frac{dy}{dx} + Py = Q$

Where, $P = \sec x \; and \; Q = \tan x$

Calculating integration factor,

$$I.F = e^{\int Pdx}$$

$$I.F = e^{\int \sec x dx}$$

$$I.F = e^{\log(\sec x + \tan x)}$$

$$I.F = \sec x + \tan x$$

∴ Solution of given differential equation is given by relation,

$$y(I.F.) = \int (Q \times I.F.)dx + C$$

$$y(\sec x + \tan x) = \int \tan x(\sec x + \tan x)dx + C$$

$$y(\sec x + \tan x) = \int \sec x \tan x \; dx + \int \tan^2 x \; dx + C$$

$$y(\sec x + \tan x) = \sec x + \int (sec^2 x - 1) \; dx + C$$

$$y(\sec x + \tan x) = \sec x + \tan x - x + C$$

5.

$$\cos^2 x\frac{dy}{dx}+y=\tan x\left(0\le x\le\frac{\pi}{2}\right)$$

Ans - On rearranging the given equation

$$\frac{dy}{dx}+\sec^2 x.y=\sec^2 x\tan x$$

Given equation is in the form of $\frac{dy}{dx}+Py=Q$

Where, $P=\sec^2 x$ and $Q=\sec^2 x\tan x$

Calculating integration factor

$I.F.=e^{\int Pdx}$

$I.F.=e^{\int \sec^2 xdx}$

$I.F.=e^{\tan x}$

$\therefore$ Solution of given differential equation is given by relation,

$y(I.F.)=\int (Q\times I.F.)\,dx+C$

$ye^{\tan x}=\int \tan x\sec^2 xe^{\tan x}dx+C$

Put $\tan x=t$

Differentiating wr.t. t

$\sec^2 xdx=dt$

$\Rightarrow ye^{\tan x}=\int te^t dt+C$

$ye^{\tan x}=te^t-\int e^t dt+C$

$ye^{\tan x}=te^t-e^t+C$

Substituting the value of t

$ye^{\tan x}=\tan xe^{\tan x}-e^{\tan x}+C$

$y=\tan x-1+Ce^{-\tan x}$

6.

$$x\frac{dy}{dx}+2y=x^2\log x$$

Ans - On rearranging given equation can be written as

$$\frac{dy}{dx}+\frac{2}{x}y=x\log x$$

Given equation is in the form of $\frac{dy}{dx}+Py=Q$

where, $P=\frac{2}{x}$ and $Q=x\log x$

Calculating integration factor

$$I.F = e^{\int Pdx}$$

$$I.F = e^{\int \frac{2}{x}dx}$$

$$I.F = e^{\log x^2}$$

$$I.F. = x^2$$

∴ Solution of given differential equation is given by relation.

$$y(I.F.)=\int (Q\times I.F.)\,dx+C$$

$$yx^2=\int x\log x.x^2dx+C$$

$$yx^2=\int x^3\log x dx+C$$

Using integration by parts,

$$yx^2=\log x\int x^3dx-\int\left(\frac{d}{dx}(\log x)\int x^3dx\right)dx+C$$

$$yx^2=\frac{x^4\log x}{4}-\int\left(\frac{1}{x}.\frac{x^4}{4}\right)dx+C$$

$$yx^2=\frac{x^4\log x}{4}-\frac{1}{4}.\frac{x^4}{4}+C$$

$$yx^2=\frac{x^4\log x}{4}-\frac{x^2}{16}+Cx^{-2}$$

$$y=\frac{x^2}{16}(4\log x-1)Cx^{-2}$$

7.

$$\boldsymbol{x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x}$$

Ans - On rearranging given equation can be written as,

$$\frac{dy}{dx}+\frac{y}{\log x}=\frac{2}{x^2}$$

Given equation is in the form of $\frac{dy}{dx}+Py=Q$

Where, $P=\frac{1}{x\log x}$ and $Q=\frac{2}{x^2}$

Calculating integration factor,

$$I.F.=e^{\int Pdx}$$

$$I.F.=e^{\log(\log x)}$$

$$I.F.=\log x$$

∴ Solution of given differential equation is given by relation.

$$y(I.F.)=\int(Q\times I.F.)\,dx+C$$

$$y(I.F.)=\int\left(\frac{2}{x^2}\log x\right)dx+C$$

Using integration by parts,

$$y\log x=2\left[\log x\int\frac{1}{x^2}dx-\int\left(\frac{d}{dx}(\log x)\int\frac{1}{x^2}dx\right)dx\right]+C$$

$$y=\log x=2\left[\log x\left(-\frac{1}{x}\right)-\int\left(\frac{1}{x}\left(-\frac{1}{x}\right)\right)dx\right]+C$$

$$y\log x=2\left[\log x\left(-\frac{1}{x}\right)+\int\frac{1}{x^2}dx\right]+C$$

$$y=\log x=-\frac{2}{x}\left[\log x\left(-\frac{1}{x}\right)+\frac{1}{x}\right]+C$$

$$y\log x=-\frac{2}{x}(1+\log x)+C$$

8.

$$\boldsymbol{(1+x^2)dy+2xydx=\cot x dx}$$

Ans - On rearranging given equation can be written as,

$$\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{\cot x}{1+x^2}$$

Given equation is in the form of $\frac{dy}{dx}+Py=Q$

Where, $P=\frac{2x}{1+x^2}$ and $Q=1+x^2$

Calculating integration factor,

$I.F.=e^{\int Pdx}$

$I.F.=e^{\int \frac{2x}{1+x^2}dx}$

$I.F.=e^{\log(1+x^2)}$

$I.F.=1+x^2$

∴ Solution of given differential equation is given by relation

$y(I.F.)=\int (Q\times I.F.)\,dx+C$

$$y(1+x^2)=\int \frac{\cot x}{1+x^2}\times(1+x^2)dx+C$$

$$y(1+x^2)=\int \cot x\,dx+C$$

$y(1+x^2)=\log(\sin x)+C$

9.

$$\boldsymbol{x\frac{dy}{dx}+y-x+xy\cot x=0}$$

Ans - On rearranging given equation can be written as

$$\frac{dy}{dx}+\left(\frac{1}{x}+\cot x\right)y=1$$

Given equation is in the form of $\frac{dy}{dx}+Py=Q$

Where, $P=\frac{1}{x}+\cot x$ and $Q=1$

Calculating integration factor,

$$I.F.=e^{\int P dx}$$

$$I.F.=e^{\int\left(\frac{1}{x}+\cot x\right)dx}$$

$$I.F.=e^{\log x+\log\sin x}$$

$$I.F.=x\sin x$$

∴ Solution of given differential equation is given by relation,

$$y(I.F.)=\int(Q\times I.F.)\,dx+C$$

$$yx\sin x=\int(1.x\sin x)dx+C$$

$$yx\sin x=x\int\sin x dx-\int\left(\frac{d}{dx}(x)\int\sin x dx\right)dx+C$$

$$yx\sin x=x(-\cos x)-\int(-\cos x)dx+C$$

$$yx\sin x=-x\cos x+\sin x+C$$

$$y=\frac{-x\cos x}{x\sin x}+\frac{\sin x}{x\sin x}+\frac{C}{x\sin x}$$

$$y=-\cot x+\frac{1}{x}+\frac{C}{x\sin x}$$

This is required differential equation.

10.

$$(\boldsymbol{x}+\boldsymbol{y})\frac{\boldsymbol{dy}}{\boldsymbol{dx}}=\mathbf{1}$$

Ans - On rearranging the given eguation can be written as

$$\frac{dx}{dy} - x = y$$

Given equation is in the form of $\frac{dx}{dy} + Px = Q$

Where, $P = -1$ and $Q = y$

Calculating integration factor

$$I.F. = e^{\int -1 dy}$$

$$I.F. = e^{-y}$$

∴ Solution of given differential equation is given by relation

$$x(I.F.) = \int (Q \times I.F.)dy + C$$

$$xe^{-y} = \int (ye^{-y})dy + C$$

$$xe^{-y} = y\int e^{-y}dy - \int \left(\frac{d}{dy}(y)\int e^{-y}dy\right) dy + C$$

$$xe^{-y} = -ye^{-y} - \int (-e^{-y})dy + C$$

$$xe^{-y} = -ye^{-y} - e^{-y} + C$$

$$x = -y - 1 + Ce^{y}$$

$$x + y + 1 = Ce^{y}$$

11.

$$\boldsymbol{ydx + (x - y^2)dy = 0}$$

Ans - On rearranging the given equation can be written as

$$\frac{dx}{dy} + \frac{x}{y} = y$$

Given guatio sinth fom of $\frac{dx}{dy} + Px = Q$

Where, $P = \frac{1}{y}$ and $Q = y$

Calculating integration factor,

$I.F. = e^{\int P dy}$

$\text{I.F} = e^{\int \frac{1}{y} dy}$

$I.F = e^{\log y}$

$I.F. = y$

$\Rightarrow x(I.F.) = \int (Q \times I.F.) dy + C$

$xy = \int (y.y)\, dy + C$

$xy = \int y^2 dy + C$

$xy = \frac{y^3}{3} + C$

12.

$$(\boldsymbol{x + 3y^2})\frac{\boldsymbol{dy}}{\boldsymbol{dx}} = \boldsymbol{y}\,(\boldsymbol{y > 0})$$

Ans - On rearranging the given equation canbe written as

$\frac{dx}{dy} - \frac{x}{y} = 3y$

Given equation is in the form of $\frac{dx}{dy} + Px = Q$

Where, $P = -\frac{1}{y}$ and $Q = 3y$

Calculating integration factor,

$I.F. = e^{\int P dy}$

$\text{I.F} = e^{\int \left(-\frac{1}{y}\right) dy}$

$I.F = e^{\log\left(\frac{1}{y}\right)}$

$I.F. = \frac{1}{y}$

∴ Solution of qiven differential equation is qiven by relation

$x(I.F.) = \int (Q \times I.F.)dy + C$

$x\left(\frac{1}{y}\right) = \int \left(3y \times \frac{1}{y}\right) dy + C$

$\frac{x}{y} = \int 3dy + C$

$\frac{x}{y} = 3y + C$

$x = 3y^2 + Cy$

For each of the differential equations given in Exercises 13 to 15, find a particular solution satisfying the given condition:

13.

$$\frac{dy}{dx} + 2y\tan x = \sin x; y = 0 \text{ when } x = \frac{\pi}{3}$$

Ans - Given equation is in the form of $\frac{dy}{dx} + Py = Q$

Where, P = 2 P = 2 P = 2tan$x$ and Q = sin$x$

Calculating integration factor

$\text{I.F} = e^{\int Pdx}$

$\text{I.F} = e^{\int 2\tan x dx}$

$\text{I.F} = e^{\log(\sec^2 x)}$

$I.F. = \sec^2 x$

∴ Solution of given differential equation is given by relation,

$y(I.F.) = \int (Q \times I.F.)\, dx + C$

$y\sec^2 x = \int (\sin x.\sec^2 x) dx + C$

$y\sec^2 x = \int (\sec x \tan x) dx + C$

$y\sec^2 x = \sec x + C \ldots\ldots (1)$

Now $y = 0$ when $x = \frac{\pi}{3}$

$0 \times \sec^2\left(\frac{\pi}{3}\right) = \sec\left(\frac{\pi}{3}\right) + C$

$C = -2$

Substituting the value of C = –2 in equation (1)

$y\sec^2 x = \sec x - 2$

$y = \cos x - 2\sec^2 x$

14.

$$\boldsymbol{(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}; y = 0 \text{ when } x = 1}$$

Ans - On rearranging the given equation can be written as,

$$\frac{dy}{dx} + \frac{2xy}{1 + x^2} = \frac{1}{(1 + x^2)^2}$$

Given equation is in the form of $\frac{dy}{dx} + Py = Q$

Where, $P = \frac{2xy}{1+x^2}$ and $Q = \frac{1}{(1+x^2)^2}$

Calculating integration factor,

$\text{I.F} = e^{\int Pdx}$

$\text{I.F} = e^{\int \frac{2xy}{1+x^2} dx}$

$\text{I.F} = e^{\log(1+x^2)}$

$I.F. = 1 + x^2$

∴ Solution of given differential equation is given by relation,

$y(I.F.) = \int (Q \times I.F.)\, dx + C$

$y(1+x^2) = \int \left(\frac{1}{(1+x^2)^2}.(1+x^2)\right) dx + C$

$y(1+x^2) = \int \frac{1}{1+x^2} dx + C$

$y(1+x^2) = \tan^{-1}x + C \ldots\ldots(1)$

Now, $y = 0$ when $x = 1$

$0 = \tan^{-1}1 + C$

$C = -\frac{\pi}{4}$

Substituting the value of $C = \frac{\pi}{4}$ in equation (1).

$y(1+x^2) = \tan^{-1}x - \frac{\pi}{4}$

15.

$\boldsymbol{\frac{dy}{dx} - 3y\cot x = \sin 2x; y = 2 \text{ when } x = \frac{\pi}{2}}$

Ans - Given equation is in the form of $\frac{dy}{dx} + Py = Q$

Where, $P = -3\cot x$ and $Q = \sin 2x$

Calculating integration factor,

$I.F = e^{\int P dx}$

$I.F. = e^{\int -3\cot x dx}$

$I.F. = e^{-3\log(\sin x)}$

$I.F. = \frac{1}{\sin^3 x}$

∴ Solution of given differential equation is given by relation

$y(I.F.) = \int (Q \times I.F.)\, dx + C$

$y.\frac{1}{\sin^3 x} = \int \left(\sin 2x.\frac{1}{\sin^3 x}\right) dx + C$

$y.\text{cosec}^3x = 2\int(\cot x\,\text{cosec}x)dx + C$

$y.\text{cosec}^3x = -2\text{cosec}x + C$

$y = -\frac{2}{\text{cosec}^2x} + \frac{C}{\text{cosec}^3x}$

$y = -2\sin^2x + C\sin^3x \ldots\ldots(1)$

Now, $y = 2$ when $x = \frac{\pi}{2}$

$2 = -2\sin^2\left(\frac{\pi}{2}\right) + C\sin^3\left(\frac{\pi}{2}\right)$

$2 = -2 + C$

$C = 4$

Substituting the value of $C = 4$ in equation (1)

$y = -\sin^2 x + 4\sin^3 x$

$y = 4\sin^3 x - 2\sin^2 x$

16.

Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

Ans - Let $f(x, y)$ be the curve passing through the origin.

At point (x, y) the slope of curve will be $\frac{dy}{dx}$.

According to the question,

$\frac{dy}{dx} - y = x$

Given equation is in the form of $\frac{dx}{dy} + Px = Q$

Where, $P = -1$ and $Q = x$

Calculating integration factor

$I.F. = e^{\int -1dx}$

$I.F. = e^{-x}$

$\therefore$ Solution of given differential equation is given by relation,

$x(I.F.) = \int (Q \times I.F.)dy + C$

$ye^{-x} = \int (xe^{-x})dx + C$

$ye^{-x} = x\int e^{-x}dy - \int \left(\frac{d}{dx}(x)\int e^{-x}dx\right)dx + C$

$ye^{-x} = -xe^{-x} - \int (-e^{-x})dx + C$

$ye^{-x} = -xe^{-x} - e^{-x} + C$

$y = -x - 1 + Ce^{x}$

$x + y + 1 = Ce^{x} \ldots\ldots(1)$

As, equation is passing through the origin.

$\therefore 0 + 0 + 1 = Ce^{0}$

$1 = C$

Substituting the value of $1 = C$ in equation 1.

$x + y + 1 = e^{x}$

This is required differential equation of curve passing through origin.

17.

Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

Ans - Let $f(x, y)$ be the cuve passing through the origin.

At point (x, y) the slope of curve will be $\frac{dy}{dx}$

According to the question,

$$\frac{dy}{dx} + 5 = x + y$$

$$\frac{dy}{dx} - y = x - 5$$

Th given equation is in the form of $\frac{dx}{dy} + Px = Q$

Where, $P = -1$ and $Q = x - 5$

Calculating integration factor,

$$I.F. = e^{\int -1dx}$$

$$I.F. = e^{-x}$$

∴ Solution of given differential equation is given by relation

$$x(I.F.) = \int (Q \times I.F.)dy + C$$

$$ye^{-x} = \int (x - 5)e^{-x}dx + C$$

$$ye^{-x} = (x - 5)\int e^{-x}dx - \int \left(\frac{d}{dx}(x - 5)\int e^{-x}dx\right)dx + C$$

$$ye^{-x} = (5 - x)e^{-x} + \int e^{-x}dx + C$$

$$ye^{-x} = (5 - x)e^{-x} - e^{-x} + C$$

$$ye^{-x} = (4 - x)e^{-x} + C$$

$$y = 4 - x + Ce^{x}$$

$$y + x - 4 = Ce^{x} \ldots\ldots(1)$$

As equation is passing through (0, 2)

$0 + 2 - 4 = Ce^0$

$-2 = C$

Substituting the value of $-2 = C$ in equation 1

$y + x - 4 = -2e^x$

This the required equation of curve passing through (0, 2).

18.

The integrating Factor of the differential equation $x\frac{dx}{dx} - y = 2x^x$ is

(A) e^{-x} **(B) e^{-y}** **(C) $\frac{1}{x}$** **(D) x**

Ans – On rearranging the given equation can be written as,

$\frac{dy}{dx} - \frac{y}{x} = 2x$

Given equation is in the form of $\frac{dy}{dx} + Py = Q$

Where, $P = -\frac{1}{x}$ and $Q = 2x$

Calculating integration factor

$I.F. = e^{\int P dx}$

$I.F. = e^{\int -\frac{1}{x} dx}$

$I.F. = e^{-\log x}$

$I.F. = e^{\log \frac{1}{x}}$

$I.F. = \frac{1}{x}$

Hence, the correct answer is option C.

19.

The Integrating Factor of the differential equation

$(1-y^2)\frac{dx}{dy}+yx=ay(-1<y<1)$**is**

(A) $\frac{1}{y^2-1}$ (B) $\frac{1}{\sqrt{y^2-1}}$ (C) $\frac{1}{1-y^2}$ (D) $\frac{1}{\sqrt{1-y^2}}$

Ans - On rearranging the given equation can be written as

$$\frac{dx}{dy}+\frac{xy}{1-y^2}=\frac{ay}{1-y^2}$$

Given equation is in the form of $\frac{dx}{dy}+Px=Q$

Where, $P=\frac{y}{1-y^2}$ and $Q=\frac{ay}{1-y^2}$

Calculating integration factor

$$I.F.=e^{\int P dy}$$

$$I.F.=e^{\int \frac{y}{1-y^2}dy}$$

$$I.F.=e^{\log\left(\frac{1}{\sqrt{1-y^2}}\right)}$$

$$I.F.=\frac{1}{\sqrt{1-y^2}}$$

Hence, the correct answer is option D.