NCERT Solutions for Class 12 Maths Chapter 8 – Application of Integrals

Exercise 8.1

1.

Find the area of the region bounded by the ellipse
$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

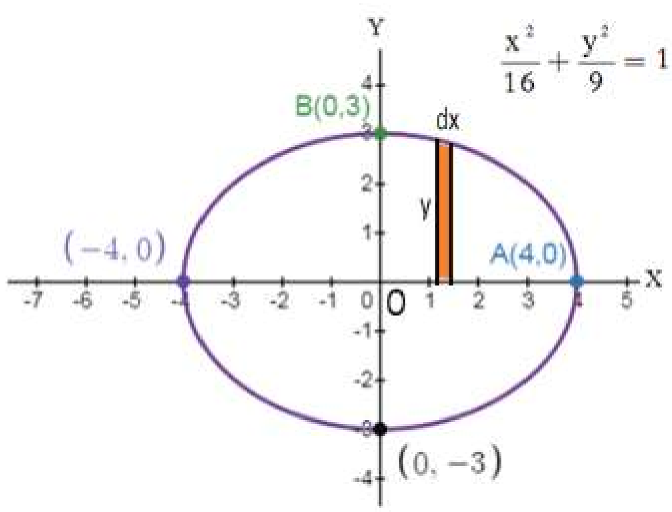
Ans – We can rewrite the given equation as

$$\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$$

Point of intersection between ellipse and x-axis are at $x=\pm 4$.

Point of intersection between ellipse and y-axis are at $y=\pm 3$.

⇒ Given equation can be represented as,



The equation stays the same if we change x to −x or y to y.

⇒ Ellipse is symmetrical about x-axis and y-axis.

Area bounded by ellipse $=4\times$ Area of OABO

Area of $OABO = \int_0^4 y dx$

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$$\Rightarrow \frac{y^2}{9}=1-\frac{x^2}{16}$$

$$\Rightarrow y^2=9\left(1-\frac{x^2}{16}\right)$$

$$\Rightarrow y=3\sqrt{1-\frac{x^2}{16}}$$

Area of OABO $= \int_0^4 3\sqrt{1-\frac{x^2}{16}}dx$

Area of OABO $= \frac{3}{4}\int_0^4 \sqrt{16-x^2}dx$

Area of OABO $= \frac{3}{4}\int_0^4 \sqrt{4^2-x^2}dx$

Apply the formula $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$ in above equation

Area of OABO $= \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

Area of OABO $= \frac{3}{4}[0+8\sin^{-1}(1)-0-8\sin^{-1}(0)]_0^4$

Area of OABO $= \frac{3}{4}\left(\frac{8\pi}{2}\right)$

Area of OABO $= 3\pi$

$\therefore$ Area bounded by ellipse $= 4\times 3\pi = 12\pi$ sq. units

2.

Find the area of the region bounded by the ellipse
$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

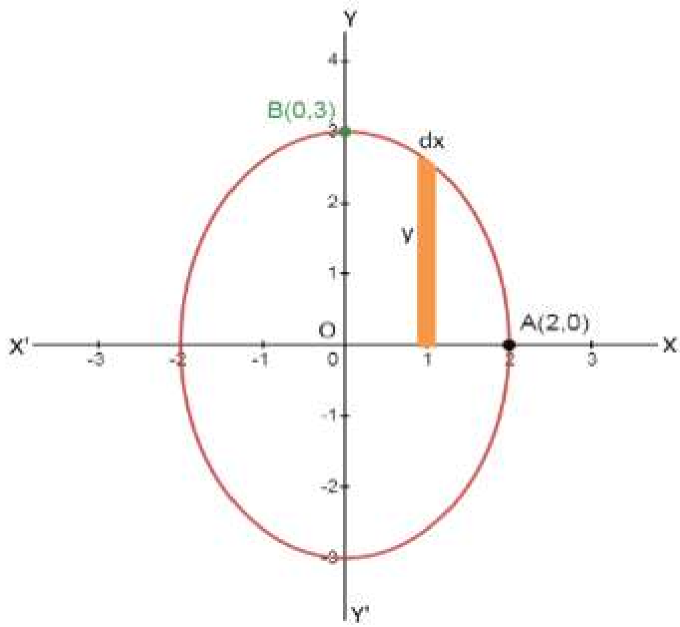
Ans – We can rewrite the given equation as

$$\frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$$

Point of intersection between ellipse and x-axis are at x=$\pm$2.

Point of intersection between ellipse and y-axis are at y=$\pm$3.

$\Rightarrow$ Given equation can be represented as,



Area of $OABO = \int_0^2 ydx$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1 \ldots.(1)$$

$$\Rightarrow \frac{y^2}{9} = 1 - \frac{x^2}{4}$$

$$\Rightarrow y^2 = 9\left(1 - \frac{x^2}{4}\right)$$

$$\Rightarrow y = 3\sqrt{1 - \frac{x^2}{4}}$$

$$\Rightarrow y = \frac{3}{2}\sqrt{4 - x^2} \ \ldots..(2)$$

Area of OABO $= \int_0^2 \frac{3}{2}\sqrt{4-x^2}dx$ using equation (2) in (1)

Area of OABO $= \frac{3}{2}\int_0^2 \sqrt{2^2-x^2}\,dx$

Apply the formula $\int \sqrt{a^2-x^2}dx$

$$= \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} \text{ in above equation}$$

Area of OABO $= \frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

Area of OABO $= \frac{3}{2}\left[\frac{2\pi}{2}\right]$

Area of OABO $= \frac{3\pi}{2}$

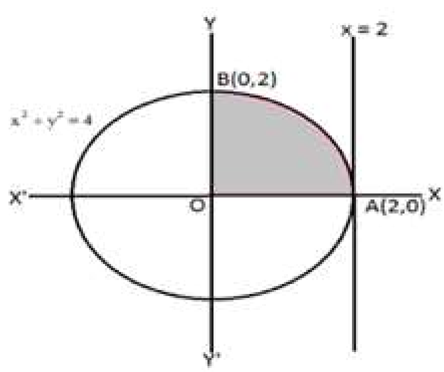
$\therefore$ Area bound by the ellipse $= 4 \times \frac{3\pi}{2} = 6\pi$ sq. units

3.

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ ad the lines $x = 0$ and $x = 2$ is

(A) π **(B) $\frac{\pi}{2}$** **(C) $\frac{\pi}{3}$** **(D) $\frac{\pi}{4}$**

Ans –



Area of $OAB = \int_0^2 y\,dx$

Since, $\mathbf{x}^2 + y^2 = 4$ [Equation of circle]

$y^2 = 4 - \mathbf{x}^2$

$y = \sqrt{4-\mathbf{x}^2}$

$\Rightarrow$ Area of $OAB = \int_0^2 \sqrt{4 - x^2}\, dx$

Area of $OAB = \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

Area of $OAB = \left[\frac{2}{2}\sqrt{4 - 2^2} + \frac{4}{2}\sin^{-1}\frac{2}{2} - \left(\frac{0}{2}\sqrt{4 - 0^2} + \frac{4}{2}\sin^{-1}\frac{0}{2}\right)\right]$

Area of $OAB = [\sqrt{4 - 4} + 2\sin^{-1}1 - 0]$

Area of $OAB = \left[2 \times \frac{\pi}{2}\right]$

Area of $OAB = \pi$ sq. units

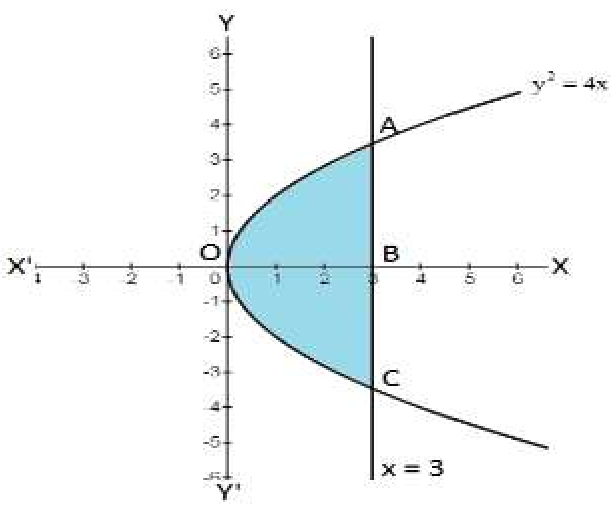
Hence, correct answer is option A.

4.

Area of the region bounded by the curve $y^2 = 4x$, y - axis and the line $y = 3$ is

(A) 2 **(B) $\frac{9}{4}$** **(C) 3** **(D) $\frac{9}{2}$**

Ans –



Area of $OAB = \int_0^3 x\, dy$

Area of $OAB = \int_0^3 \frac{y^2}{4} dy$

Area of $OAB = \frac{1}{4}\int_0^3 y^2\, dy$

Area of $OAB = \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$

Area of $OAB = \frac{1}{4}\left[\frac{3^3}{3} - 0\right]$

Area of $OAB = \frac{1}{4}\left[\frac{27}{3}\right]$

Area of $OAB = \frac{1}{4}[9]$

Area of $OAB = \frac{9}{4}$ square unis

Hence, correct answer is option B.

Miscellaneous Exercise

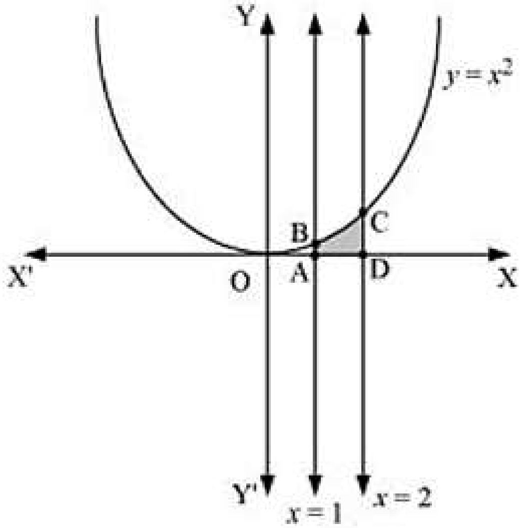
1.

Find the area under the given curves and given lines.

(i) $y = x^2, x = 1, x = 2$ and x - axis

(ii) $y = x^4, x = 1, x = 5$ and x - axis

Ans – (i)



Area ADCBA $= \int_1^2 y \, dx$

We know $y = x^2$

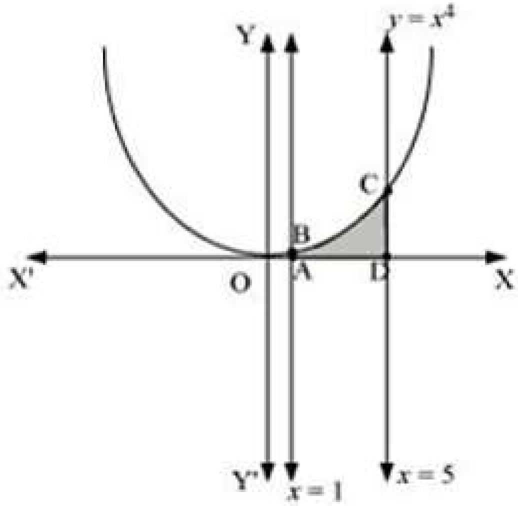
$\Rightarrow$ Area ADCBA $= \int_1^2 x^2 \, dx$

Substituting the limits,

$= \frac{8}{3} - \frac{1}{3}$

$= \frac{7}{3}$ units

(ii)



Area of ADCBA= $\int_1^5 x^4 dx$

Integrating using the power rule,

$$= \left[\frac{x^5}{5}\right]_1^5$$

Substituting the limits,

$$= \frac{(5)^5}{5} - \frac{1}{5}$$

$$= (5)^4 - \frac{1}{5}$$

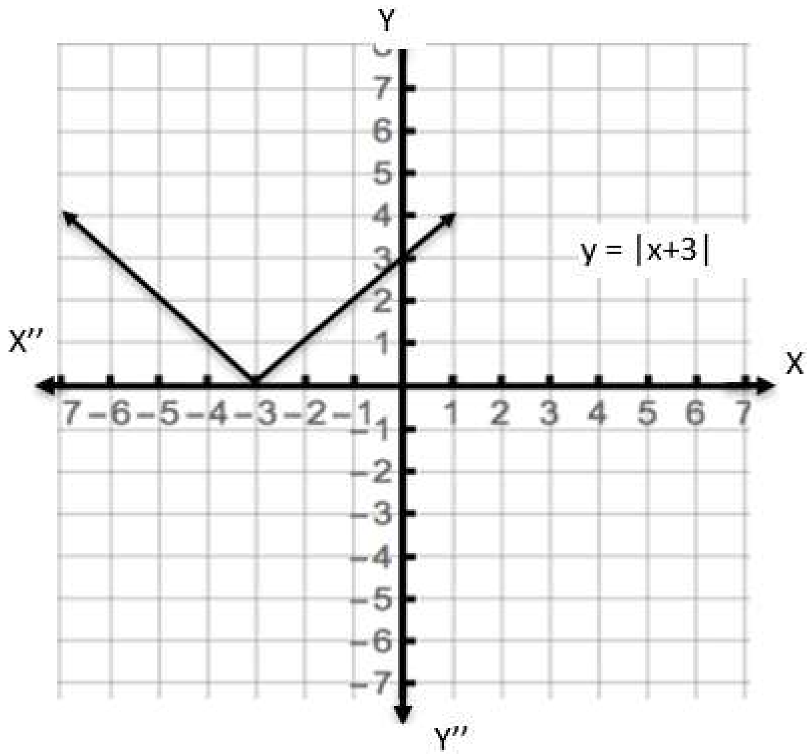
$$= 625 - \frac{1}{5} = 624.8 \text{ units}$$

2.

Sketch the graph of y $= |x + 3|$ and evaluate $\int_{-6}^{0} |x + 3|\, dx$

Ans -

x	-6	-5	-4	-3	-2	-1	0
y	3	2	1	0	1	2	3



$(\mathbf{x}+3) \le 0$ for $-6 \le \mathbf{x} \le -3$

$(\mathbf{x}+3) \ge 0$ for $-3 \le \mathbf{x} \le 0$

$$\Rightarrow \int_{-6}^{0} |(x+3)|dx = -\int_{-6}^{3} (x+3)\,dx + \int_{-3}^{0} (x+3)\,dx$$

Integrating using the power rule

$$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

Substituting the limits

$$= -\left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right]$$

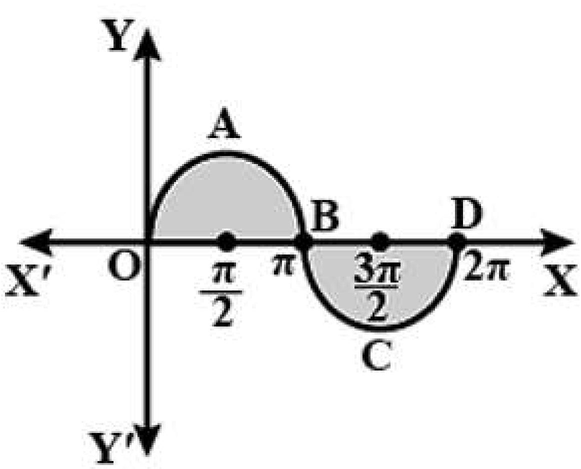
$$= -\left[-\frac{9}{2}\right] - \left[-\frac{9}{2}\right]$$

$$= 9$$

3.

Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Ans - Area = Area OABO + Area BCDB



Area Bounded by Curve $y = \sin x$

$$= \int_0^{\pi} \sin x \, dx + \left| \int_{\pi}^{2\pi} \sin x \, dx \right|$$

$$= [-\cos x]_0^{\pi} + [-\cos x]_{\pi}^{2\pi}$$

Substituting the limits,

$$= [-\cos \pi + \cos 0] + |-cos 2\pi + cos\pi|$$

Simplifying,

$=1+1+|(-1-1)|$

$=2+|-2|$

$=2+2$

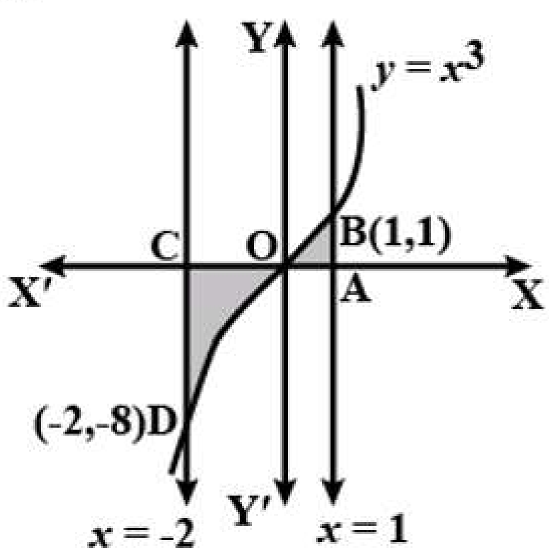
$= 4$ units

4.

Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

A. -9 **B.** $-\frac{15}{4}$ **C.** $\frac{15}{4}$ **D.** $\frac{17}{4}$

Ans -



As shown in the diagram required area $= \int_{-2}^{1} \text{ydx}$

$$= \int_{-2}^{1} \mathbf{x}^3 \, dx$$

Integrating using the power rule

$$= \left[\frac{x^4}{4}\right]_{-2}^{1}$$

Substituting the limits,

$$= \left[\frac{1}{4} - \frac{(-2)^4}{4}\right]$$

$$= \left(\frac{1}{4} - 4\right)$$

$$= -\frac{15}{4} \text{ units}$$

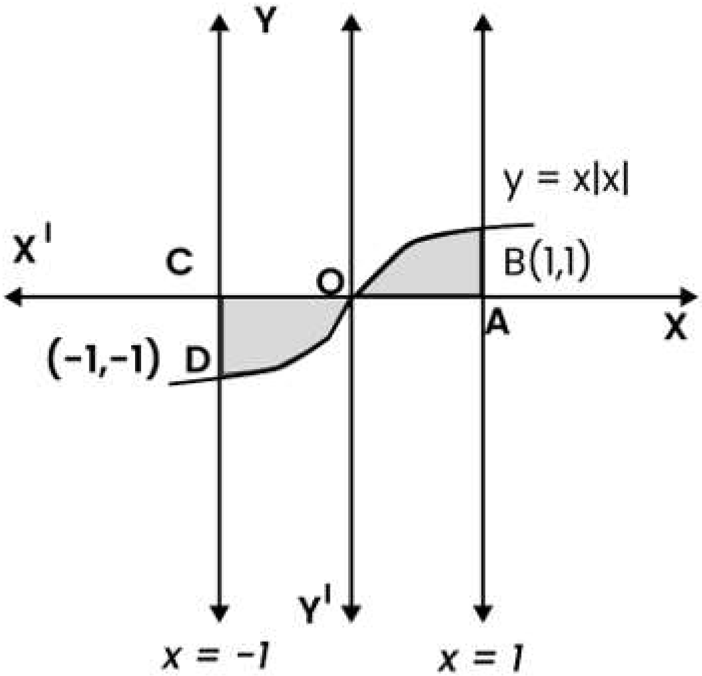
Hence, correct answer is option B.

5.

The area bounded by the curve $y = x|x|$, x - axis and the ordinate $x = 1$ and $x = -1$ is given by (Hint $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$)

A. 0 **B. $\frac{1}{3}$** **C. $\frac{2}{3}$** **D. $\frac{4}{3}$**

Ans -



Required area $= \int_{-1}^{1} y dx$

$= \int_{-1}^{1} x|x| dx$

$= \int_{-1}^{0} x^2 dx + \int_{-1}^{1} x^2 dx$

Integrating using the power rule

$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$

Substituting the limits,

$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$

$= \frac{2}{3}$ units

Hence, correct answer is option C.