NCERT Solutions for Class 12 Maths Chapter 4 – Determinants

Exercise 4.1

1.

Evaluate the determinant: $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

Ans - Solving the determinants $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$ we have:

$\Rightarrow \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2(-1) - 4(-5)$

$\Rightarrow \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = -2 + 20$

$\Rightarrow \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 18$

2.

Evaluate the determinants.

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

Ans – (i) Solving the determinant

$$\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

We have: $\Rightarrow \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = (\cos\theta)(\cos\theta) - (-\sin\theta)(\sin\theta)$

$\Rightarrow \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta$

we know, $\cos^2\theta + \sin^2\theta = 1$

$\therefore \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = 1$

(ii) Solving the determinant

$$\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$$

$\Rightarrow \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = (x^2 - x + 1)(x + 1) - (x - 1)(x + 1)$

$\Rightarrow \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = x^3 - x^2 + x - x^2 - x + 1 - (x^2 - 1)$

So, $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = x^3 + 1 - x^2 + 1$

$\therefore \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = x^3 - x^2 + 2$

3.

IF $A = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$, then show that $|2A| = 4|A|$

Ans - Given $A = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$.

Multiplying A by 2, we get

$2A = 2A = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$

$\Rightarrow A = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$

$\Rightarrow 2A = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$

$\therefore$ L.H.S $= |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$

$\Rightarrow 2 \times 4 - 4 \times 8 \Rightarrow |2A| = 8 - 32$

$\therefore |2A| = -24$

Determinant of A is $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$

$\Rightarrow |A| = 2 - 8$

$\therefore |A| = -6$

R.H.S is given as $4|A|$.

$\therefore 4|A| = 4 \times (-6) = -24.$

$\Rightarrow$ L.H.S $=$ R.H.S $\therefore |2A| = 4|A|.$

4.

If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$.

Ans - Given, $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

Determining the value of determinant A, by expanding among the first column, i.e., C1, we get:

$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$

$\Rightarrow |A| = 1(4 - 0) - 0 + 0 \Rightarrow |A| = 4$

$\Rightarrow 3A = 3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

$\therefore 27|A| = 27 \times 4 = 108 \ldots\ldots (1).$

The value of $|3A|$ is obtained as: $\Rightarrow 3A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$

$\therefore |3A| = 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$

$\Rightarrow 3(36 - 0) + 0 + 0 \Rightarrow |3A| = 3 \times 36$

$\Rightarrow |3A| = 108 \ldots.. (2)$

From equation (1) and (2), we get $|3A| = 27|A|$

Hence proved.

5.

Evaluate the determinants

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

(iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Ans - (i) Let $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Determining the value of A by expanding along the 2nd row, we will get:

$$|A| = -0 \begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0 \begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix}$$

$$\Rightarrow |A| = (-15 + 3) \therefore |A| = -12$$

(ii) Let $A = \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

Determining the value of A by expanding along the 1st row, we have:

$$|A| = 3 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$

$$\Rightarrow |A| = 3(1 + 6) + 4(1 + 4) + 5(3 - 2)$$

$$\Rightarrow |A| = 31 + 20 + 5 \therefore |A| = 46$$

(iii) Let $A = \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

Determining the value of A by expanding along the 1st row, we have:

$$|A| = 0\begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2\begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$$

$\Rightarrow |A| = 0(9) - (-6) + 2(-3) \therefore |A| = 0$

(iv) Let $A = \begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Determining the value of A by expanding along the 1st column, we have:

$$|A| = 2\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - 0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 3\begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix}$$

$\Rightarrow |A| = 2(-5) - 0 + 3(5)$

$\Rightarrow |A| = -10 + 5 \therefore |A| = 5$

6.

If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find $|A|$.

Ans - Given $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

Determining the value of A by expanding along the 1st row, we have:

$$|A| = 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

$\Rightarrow |A| = 1(-9 + 12) - 1(-18 + 15) - 2(8 - 5)$

$\Rightarrow |A| = 3 + 3 - 6 \therefore |A| = 0$

7.

Find the values of x, if

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Ans – (i) Given $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

Solving it, we have:

$(2 \times 1) - (5 \times 4) = (2x - x) - (6 \times 4)$

$\Rightarrow 2 - 20 = 2x^2 - 24$

$\Rightarrow -18 + 24 = 2x^2$

$\Rightarrow 3 = x^2$

Applying square root on both sides we get,

$x = \pm\sqrt{3}$

(ii) Given $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Solving it, we have:

$(2 \times 5) - (3 \times 4) = (x \times 5) - (3 \times 2x)$

$\Rightarrow 10 - 12 = 5x - 6x$

$\Rightarrow -2 = -x$

Multiplying by (-1) on both the sides we get,

$x = 2.$

8.

If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

(A) 6   (B) ± 6 (C) -6 (D) 0

Ans - Given $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

Solving it, we have:

$x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow x^2 = 36$

Applying square root on both the sides we get,

$x = \pm 6$

$\Rightarrow$ Option B is the correct answer.

Exercise 4.2

1.

Find the area of the triangle with vertices at the point given in each of the following:

(i) $(1, 0), (6, 0), (4, 3)$

(ii) $(2, 7), (1, 1), (10, 8)$

(iii) $(2, 3), (3, 2), (-1, -8)$

Ans – (i) Given vertices, $(1, 0), (6, 0), (4, 3)$. Thus the area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2}[1(0 + 3) - 0(6 - 4) + 1(18 - 0)]$$

$$\Rightarrow \Delta = \frac{1}{2}[-3 + 18]$$

$$\Rightarrow \Delta = \frac{15}{2} \text{ Square units}$$

$\therefore$ Area of the triangle with vertices $(1, 0), (6, 0), (4, 3)$ is $\frac{15}{2}$ square units.

(ii) Given vertices, $(2, 7), (1, 1), (10, 8)$. The area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2}[2(1 - 8) - 7(1 - 10) + 1(8 - 10)]$$

$$\Rightarrow \Delta = \frac{1}{2}[2(-7) - 7(-9) + 1(-2)]$$

$$\Rightarrow \Delta = \frac{1}{2}[-16 + 63]$$

$$\Rightarrow \Delta = \frac{47}{2} \text{ Square units}$$

$\therefore$ Area of the triangle with vertices $(2, 7), (1, 1), (10, 8)$ is $\frac{47}{2}$ square units.

(iii) Ans: Given vertices, $(2, 3), (3, 2), (-1, -8)$. The area of the triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2}[-2(2+8) + 3(3+1) + 1(-34+2)]$$

$$\Rightarrow \Delta = \frac{1}{2}[-20 + 12 - 22]$$

$$\Rightarrow \Delta = -\frac{30}{2} \text{ Square units}$$

$$\Rightarrow \Delta = -15$$

∴ The area of the triangle with vertices $(2, 7), (1, 1), (10, 8)$ is $|-15| = 15$ square units.

2.

Show that points $A(a, b + c), B(b, c + a), C(c, a + b)$ are collinear.

Ans – To show the points are collinear the arra of the triangle formed by these points as vertices sholud be zero.

∴ Area of ∆ABC is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

Applying the row operation, $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \Delta = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b-a & a-b & 0 \\ c-a & a-c & 0 \end{vmatrix}$$

Taking out $(a - b)$ common from R_2 and R_3 recpectively,

$$\Rightarrow \Delta = \frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

Applying the row operation $R_3 \rightarrow R_3 + R_2$

$$\Rightarrow \Delta = \frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

We are aware that the value of a determinant is always zero when every element in a row or column is zero.

∴ $\Delta = 0$

∴ the area of the triangle formed by points A, B and C is zero.

Hence, the points $A(a, b + c)$, $B(b, c + a)$, $C(c, a + b)$ are collinear

3.

Find the values of k if area of triangle is 4 sq. units and vertices are.

(i) (k, 0), (4, 0), (0, 2)   **(ii) (−2, 0), (0, 4), (0, k)**

Ans - (i) Given vertices are $(k, 0)$, $(4, 0)$, $(0, 2)$.

We know, the area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2}[k(0-2) - 0(4-0) + 1(8-0)]$$

$$\Rightarrow \Delta = \frac{1}{2}[-2k + 8]$$

$$\therefore \Delta = k + 4$$

Since the area = 4 square units,

$$\Rightarrow -k + 4 = \pm 4$$

When $-k + 4 = -4$

$$\therefore k = 8$$

When $-k + 4 = 4$

$$\therefore k = 0$$

Hence, $k = 0, 8$.

(ii) Given vertices are $(-2, 0), (0, 4), (0, k)$.

The area of the triangle is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2}[-2(4-k)]$$

$$\Delta = k - 4$$

Since the area $= 4$ square units,

$$\Rightarrow k - 4 = \pm 4$$

When $k - 4 = -4$

$$\therefore k = 0$$

When $k - 4 = 4$

$$\therefore k = 8.$$

Hence, $k = 0, 8$.

4.

Determine the following:

(i) Find the equation of line joining (1,2) and (3,6) using determinants.

(ii) Find the equation of line joining (3,1) and (9,3) using determinants.

Ans - (i) Let us assume a point, P(x, y) on the line joining points A (1, 2) and B (3, 6).

Then, the points A, B and P are collinear.

Thus, the area of the triangle ABP will zero.

$$\therefore \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}[1(6-y) - 2(3-x) + 1(3y - 6x)] = 0$$

$$\Rightarrow 6 - y - 6 + 2x + 3y - 6x = 0$$

$$\Rightarrow 2y - 4x = 0$$

$$\Rightarrow y = 2x$$

∴The equation of the line joining the given points is y = 2x.

(ii) Let us assume a point, P(x, y) on the line joining points A(3, 1) and B (9, 3).

Then, the points A, B and P are collinear.

Thus, the area of the tringle ABP will be zero.

$$\therefore \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}[3(3-y) - 1(9-x) + 1(9y - 3x)] = 0$$

$$\Rightarrow 9 - 3y - 9 + x + 9y - 3x = 0$$

$$\Rightarrow 6x - 2x = 0$$

$$\Rightarrow x - 3y = 0$$

∴ The equation of the line joining the given points is x − 3y = 0

5.

If the area of triangle is 35 square units with vertices (2,-6),
(5,4) and (k,4). Then k is

A. 12   B. -2   C. -12,-2   D. 12,-2

Ans - Given vertices. (2, -6), (5, 4) and (k, 4)

The area of the triangle given by,

$$\Rightarrow \Delta = \frac{1}{2} \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2} [2(4-4) + 6(5-k) + 1(20-4k)]$$

$$\Rightarrow \Delta = \frac{1}{2} [50 - 10k]$$

$$\Rightarrow \Delta = 25 - 5k$$

Given, the area of the triangle is 35 square units.

$$\Rightarrow 25 - 5k = \pm 35$$

$$\Rightarrow 5(5 - k) = \pm 35$$

$$\Rightarrow 5 - k = \pm 7.$$

When $5 - k = 7$

$$\therefore k = -2.$$

When $5 - k = -7$

$$\therefore k = 12.$$

$$\Rightarrow k = 12, -2.$$

$\therefore$ Option D. $12, -2$ is the correct option.

Exercise 4.3

1.

Write Minors and Cofactors of the elements of following determinants:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Ans - (i) Given, $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

Minor of element A_{ij} is denoted by M_{ij}.

$\therefore M_{11} = 3$

$M_{12} = 0$

$M_{21} = -4$

$M_{22} = 2$

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j}M_{ij}$

$\Rightarrow A_{11} = (-1)^{1+1}M_{11}$

$A_{11} = (-1)^2(3)$

$\therefore A_{11} = 3$

Similarly,

$\Rightarrow A_{12} = (-1)^{1+2}M_{12}$

$A_{12} = (-1)^3(0)$

$\therefore A_{12} = 0$

$\Rightarrow A_{21} = (-1)^{2+1}M_{21}$

$A_{21} = (-1)^3(-4)$

$\therefore A_{21} = 4$

$\Rightarrow A_{22} = (-1)^{2+2}M_{22}$

$A_{22} = (-1)^4(2)$

$\therefore A_{22} = 2$

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = 1$

$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = 0$

$\Rightarrow A_{13} = (-1)^{1+3} M_{13} = 0$

$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = 0$

$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = 1$

$\Rightarrow A_{23} = (-1)^{2+3} M_{23} = 0$

$\Rightarrow A_{31} = (-1)^{3+1} M_{31} = 0$

$\Rightarrow A_{32} = (-1)^{3+2} M_{32} = 0$

$\Rightarrow A_{33} = (-1)^{3+3} M_{33} = 1$

(ii) Given, $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Minor of element A_{ij} is denoted by M_{ij}.

$\therefore M_{11} = d$

$M_{12} = b$

$M_{21} = c$

$M_{22} = a$

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

$\Rightarrow A_{11} = (-1)^{1+1} M_{11}$

$A_{11} = (-1)^2 d$

$\therefore A_{11} = d$

Similarly,

$\Rightarrow A_{12} = (-1)^{1+2} M_{12}$

$A_{12} = (-1)^3 (b)$

$\therefore A_{12} = -b$

$\Rightarrow A_{21} = (-1)^{2+1} M_{21}$

$A_{21} = (-1)^3 (c)$

$\therefore A_{21} = -c$

$\Rightarrow A_{22} = (-1)^{2+2} M_{22}$

$A_{22} = (-1)^4 (a)$

$\therefore A_{22} = a$

2.

Write Minors and Cofactors of the elements of following determinants:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$
(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Ans - (i) Given determinants, $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$.

Minor of element a_{ij} is denoted by M_{ij}.

$\therefore M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\Rightarrow M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$\Rightarrow M_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$\Rightarrow M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$\Rightarrow M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\Rightarrow M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\Rightarrow M_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$\Rightarrow M_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = 1$

$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = 0$

$\Rightarrow A_{13} = (-1)^{1+3} M_{13} = 0$

$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = 0$

$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = 1$

$\Rightarrow A_{23} = (-1)^{2+3} M_{23} = 0$

$\Rightarrow A_{31} = (-1)^{3+1} M_{31} = 0$

$\Rightarrow A_{32} = (-1)^{3+2} M_{32} = 0$

$\Rightarrow A_{33} = (-1)^{3+3} M_{33} = 1$

(ii) Given determinant, $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Minor of element a_{ij} is denoted by M_{ij}.

$$\therefore M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11$$

$$\Rightarrow M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$$

$$\Rightarrow M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

$$\Rightarrow M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$$

$$\Rightarrow M_{22} = \begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$$

$$\Rightarrow M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$\Rightarrow M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 + 20 = -20$$

$$\Rightarrow M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13$$

$$\Rightarrow M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$$

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j}M_{ij}$

$\Rightarrow A_{11} = (-1)^{1+1}M_{11} = 11$

$\Rightarrow A_{12} = (-1)^{1+2}M_{12} = 6$

$\Rightarrow A_{13} = (-1)^{1+3}M_{13} = 3$

$\Rightarrow A_{21} = (-1)^{2+1}M_{21} = -4$

$\Rightarrow A_{22} = (-1)^{2+2}M_{22} = 2$

$\Rightarrow A_{23} = (-1)^{2+3}M_{23} = 1$

$\Rightarrow A_{31} = (-1)^{3+1}M_{31} = -20$

$\Rightarrow A_{32} = (-1)^{3+2}M_{32} = -13$

$\Rightarrow A_{33} = (-1)^{3+3}M_{33} = 5$

3.

Using Cofactors of elements of second row, evaluate $\Delta =$
$\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Ans - Given determinant, $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Determining the minors and cofactors, we get:

$\Rightarrow M_{21} = \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = 9 - 16 = -7$

$\therefore A_{21} = (-1)^{2+1} M_{21} = 7$

$\Rightarrow M_{22} = \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$

$\therefore A_{22} = (-1)^{2+2} M_{22} = 7$

$\Rightarrow M_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$

$\therefore A_{23} = (-1)^{2+3} M_{23} = -7$

Since, Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$\therefore \Delta = a_{21} A_{21} + a_{22} A_{22} + a_{23} A_{23}$

$\Rightarrow \Delta = 2(7) + 0(7) + 1(7)$

Hence, $\Delta = 21$

4.

Using Cofactors of elements of third column, evaluate $\Delta =$
$\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

Ans - Given determinant, $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

Determining the minors and cofactors, we get:

$$\Rightarrow M_{13} = \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$$

$$\therefore A_{13} = (-1)^{1+3} M_{13} = (z - y)$$

$$\Rightarrow M_{23} = \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = z - x$$

$$\therefore A_{23} = (-1)^{2+3} M_{23} = (x - z)$$

$$\Rightarrow M_{33} = \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$$

$$\therefore A_{33} = (-1)^{3+3} M_{33} = (y - x)$$

Since, Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$$\therefore \Delta = a_{13} A_{13} + a_{23} A_{23} + a_{33} A_{33}$$

$$\Rightarrow \Delta = yz(z - y) + zx(x - z) + xy(y - x)$$

$$\Rightarrow \Delta = yz^2 - y^2 z + x^2 z - xz^2 + xy^2 - x^2 y$$

$$\Rightarrow \Delta = (x^2 z - y^2 z) + (yz^2 - xz^2) + (xy^2 - x^2 y)$$

$$\Rightarrow \Delta = (x - y)[zx + zy - z^2 - xy]$$

$$\Rightarrow \Delta = (x - y)[z(x - z) + y(z - x)]$$

This, $\Delta = (x - y)(y - z)(z - x)$.

5.

If $\Delta= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors o a_{ij}, then value of Δ is given by

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Ans - It is given that $\Delta= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

The value of Δ by expanding along first columns is obtained as,

$a_{11}(a_{22}.a_{33} - a_{23}.a_{32}) - a_{21}(a_{12}.a_{33} - a_{13}.a_{32}) + a_{31}(a_{12}.a_{23} - a_{13}.a_{22}) \dots \dots 1$

Now, the cofactor A_{ij} of element a_{ij} is given by $(-1)^{i+j}M_{ij}$, where M_{ij} is the minor. Minor is the determinant obtained by cancelling the ith row and jth columns of the original matrix.

Now for element a_{11}, the minor is $M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}.a_{33} - a_{32}.a_{32}$ and the cofactor is

$A_{11} = (-1)^{1+1}(a_{22}.a_{33} - a_{23}.a_{32}) \Rightarrow A_{11} = (a_{22}.a_{33} - a_{23}.a_{32})$.

Next for element a_{21}, the minor is $M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} =$ $a_{12}.a_{33} - a_{13}.a_{32}$ and the cofactor is

$A_{21} = (-1)^{2+1}(a_{12}.a_{33} - a_{13}.a_{32}) \Rightarrow A_{21} = -(a_{12}.a_{33} - a_{13}.a_{32})$.

Next for element a_{31}, the minor is $M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} =$ $a_{12}.a_{23} - a_{13}.a_{22}$ and the cofactor is

$A_{31} = (-1)^{3+1}(a_{12}.a_{23} - a_{13}.a_{22}) \Rightarrow A_{31} = (a_{12}.a_{23} - a_{13}.a_{22})$.

Now substituting the terms as obtained from above computation in equation 1,

$a_{11}A_{11} - a_{21}(-A_{21}) + a_{31}A_{31}$

$a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

This matches with option d.

Exercise 4.4

1.

Find the adjoint of each of the matrices. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Ans - Let A= $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

We know Cofactor of $A_{ij} = (-1)^{i+j}M_{ij}$

$A_{11} = (-1)^{1+1}M_{11} = (-1)^{1+1}M_{11}$

$\Rightarrow A_{11} = 4$

$A_{12} = (-1)^{1+2}M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = -3$

$A_{21} = (-1)^{2+1}M_{21} = (-1)^3 M_{21}$

$\Rightarrow A_{21} = -2$

$A_{22} = (-1)^{2+2}M_{22} = (-1)^4 M_{22}$

$\Rightarrow A_{22} = 1$

We know that adjoint of a matrix is the transpose of it cofactor matrix.

Thus, adj A $= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^T$

$\therefore$ adjA $= \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

2

Find the adjoint of each of the matrices. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

Ans - Let A = $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

We know Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = \begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -2(2 + 10) = -12$

$A_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$

$\Rightarrow A_{13} = \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$

Similarly

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$

$\Rightarrow A_{21} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = (-1 - 0) = 1$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$

$\Rightarrow A_{22} = \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = (1 + 4) = 5$

$A_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23}$

$\Rightarrow A_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = (0 - 2) = 2$

and

$A_{31} = (-1)^{3+1}M_{31} = (-1)^4 M_{31}$

$\Rightarrow A_{31} = \begin{vmatrix} -1 & 2 \\ 2 & 5 \end{vmatrix} = -5 - 4 = -9$

$A_{32} = (-1)^{3+2}M_{32} = (-1)^5 M_{32}$

$\Rightarrow A_{32} = \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5 - 4) = -1$

$A_{33} = (-1)^{3+3}M_{33} = (-1)^6 M_{33}$

$\Rightarrow A_{33} = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\therefore \text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T = \begin{bmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -9 & -1 & 5 \end{bmatrix}$$

3.

Verify $A(\text{adj}A) = (\text{adj}A)A = |A|I.$ $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

Ans - Given, $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$\therefore |A| = -12 - (-12)$

$\Rightarrow |A| = 0$

Hence, $|A|I = 0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow |A|I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Since, Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j}M_{ij}$

$A_{11} = (-1)^{1+1}M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = -6$

$A_{12} = (-1)^{1+2}M_{12} = (-1)^3 M_2$

$\Rightarrow A_{12} = 4$

$A_{21} = (-1)^{2+1}M_{21} = (-1)^3 M_{21}$

$\Rightarrow A_{21} = -3$

$A_{22} = (-1)^{2+2}M_{22} = (-1)^4 M_{22}$

$\Rightarrow A_{22} = 2$

Cofactor matrix is $\begin{bmatrix} -6 & 4 \\ -3 & 2 \end{bmatrix}$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$\Rightarrow \text{adj } A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

Now, multiplying A with its adjoint, we have:

$A (\text{adj } A) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

$\Rightarrow A (\text{adj } A) = \begin{bmatrix} -12 + 12 & -6 + 6 \\ 24 - 24 & 12 - 12 \end{bmatrix}$

$\Rightarrow A (\text{adj } A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Similarly

$(\text{adj } A)A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$\Rightarrow (\text{adj } A)A = \begin{bmatrix} -12 + 12 & -18 + 18 \\ 8 - 8 & 12 - 12 \end{bmatrix}$

$\Rightarrow (\text{adj } A)A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow (\text{adj } A)A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore A (\text{adj } A) = (\text{adj } A) A = |A|I$

Hence verified.

4.

Verify A(adjA) = (adjA)A = |A|I. $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

Ans - Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

$\Rightarrow |A| = 1(0-0) + 1(9+2) + 2(0-0)$

$\therefore |A| = 11$

$|A|I = 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

Since, Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = 0$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = -(9+2) = -11$

$A_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$

$\Rightarrow A_{13} = 0$

Similarly,

$A_{21} = (-1)^{2+1}M_{21} = (-1)^3M_{21}$

$\Rightarrow A_{21} = -(-3+0) = 3$

$A_{22} = (-1)^{2+2}M_{22} = (-1)^4M_{22}$

$\Rightarrow A_{22} = 3 - 2 = 1$

$A_{23} = (-1)^{2+3}M_{23} = (-1)^5M_{23}$

$\Rightarrow A_{23} = -(0+1) = -1$

and

$A_{31} = (-1)^{3+1}M_{31} = (-1)^4M_{31}$

$\Rightarrow A_{31} = 2 - 0 = 2$

$A_{32} = (-1)^{3+2}M_{32} = (-1)^5M_{32}$

$\Rightarrow A_{32} = -(-2-6) = 8$

$A_{33} = (-1)^{3+3}M_{33} = (-1)^6M_{33}$

$\Rightarrow A_{33} = 0 + 3 = 3$

Cofactor matrix is $\begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}$

We know that adjoint of a matrix is the transpose of its cofactor matrix

$$\text{adj } A = \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}^T$$

$$\Rightarrow \text{adj } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

Now, multiplying A with it adjoint, we have:

$$A(\text{adj } A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow A(\text{adj } A) = \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0+0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$\therefore A(\text{adj } A) = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Similarly, multiplying (adj A) with A, we get:

$$(\text{adj } A)A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow (\text{adj } A)A = \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0+0+0 & 0+2+9 \end{bmatrix}$$

$$\therefore (\text{adj } A)A = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Thus, A (adj A) = (adj A = |A|I)

Hence verified.

5.

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

Ans - Let A = $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

$\Rightarrow |A| = 6 + 8$

$\therefore |A| = 14$

Since, Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j}M_{ij}$

$A_{11} = (-1)^{1+1}M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = 3$

$A_{12} = (-1)^{1+2}M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = -4$

$A_{21} = (-1)^{2+1}M_{21} = (-1)^{2+1}M_{21}$

$\Rightarrow A_{21} = 2$

$A_{22} = (-1)^{2+2}M_{22} = (-1)^{2+2}M_{22}$

$\Rightarrow A_{22} = 2$

Cofactor matrix is $\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$\therefore$ adj A= $\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

Inverse of the matrix A is given by,

$A^{-1} = \dfrac{1}{|A|}$ adj A

$\therefore A^{-1} = \dfrac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

6.

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

Ans - Let A $=\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

$\Rightarrow |A| = -2 + 15$

$\therefore |A| = 13$

Since, Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j}M_{ij}$

$A_{11} = (-1)^{1+1}M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = 2$

$A_{12} = (-1)^{1+2}M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = 3$

$A_{21} = (-1)^{2+1}M_{21} = (-1)^{2+1}M_{21}$

$\Rightarrow A_{21} = -5$

$A_{22} = (-1)^{2+2}M_{22} = (-1)^{2+2}M_{22}$

$\Rightarrow A_{22} = -1$

Cofactor matrix is $\begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$\therefore$ adj A$=\begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$

Inverse of the matrix A is given by,

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\therefore A^{-1} = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

7.

Find the inverse of each of the matrices (if it exists).
$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Ans - Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

$\Rightarrow |A| = 1(10 - 0) - 2(0 - 0) + 3(0 - 0)$

$\therefore |A| = 10$

Since, Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = 10 - 0 = 10$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = -(0 + 0) = 0$

$A_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$

$\Rightarrow A_{13} = 0$

Similarly

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$

$\Rightarrow A_{21} = -(10 - 0) = -10$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$

$\Rightarrow A_{22} = 5 - 0 = 5$

$A_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23}$

$\Rightarrow A_{23} = -(0 - 0) = 0$

And

$A_{31} = (-1)^{3+1}M_{31} = (-1)^4 M_{31}$

$\Rightarrow A_{31} = 8 - 6 = 2$

$A_{32} = (-1)^{3+2}M_{32} = (-1)^5 M_{32}$

$\Rightarrow A_{32} = -(4 - 0) = -4$

$A_{33} = (-1)^{3+3}M_{33} = (-1)^6 M_{33}$

$\Rightarrow A_{33} = 2 + 0 = 2$

Cofactor matrix is $\begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$\therefore \text{adj } A = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

Inverse of the matrix A is given by,

$A^{-1} = \dfrac{1}{|A|} \text{adj } A$

$\therefore A^{-1} = \dfrac{1}{10} \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

8.

Find the inverse of each of the matrices (if it exists).
$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

Ans - Let A $= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

$\Rightarrow |A| = 1(-3 - 0) - 0 + 0$

$\therefore |A| = -3$

Since, Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = -3 - 0 = -3$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = -(-3 - 0) = 3$

$A_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$

$\Rightarrow A_{13} = 6 - 15 = -9$

Similarly

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$

$\Rightarrow A_{21} = -(0 + 0) = 0$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$

$\Rightarrow A_{22} = -1 - 0 = -1$

$A_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23}$

$\Rightarrow A_{23} = -(2 - 0) = 0$

And

$A_{31} = (-1)^{3+1} M_{31} = (-1)^4 M_{31}$

$\Rightarrow A_{31} = 0 - 0 = 0$

$A_{32} = (-1)^{3+2} M_{32} = (-1)^5 M_{32}$

$\Rightarrow A_{32} = -(0 - 0) = 0$

$A_{33} = (-1)^{3+3} M_{33} = (-1)^6 M_{33}$

$\Rightarrow A_{33} = 3 - 0 = 3$

Cofactor matrix is $\begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow \text{adj } A = \begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}^T$$

$$\therefore \text{adj } A = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Inverse of the matrix A is given by,

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

9.

Find the inverse of each of the matrices (if it exists).
$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

Ans - Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

$\Rightarrow |A| = 2(-1-0) - 1 + (4-0) + 3(8-7)$

$\Rightarrow |A| = 2(-1) - 1(4) + 3(1)$

$\therefore |A| = -3$

Since, Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = -1 - 0 = -1$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = -(4-0) = -4$

$A_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$

$\Rightarrow A_{13} = 8 - 7 = 1$

Similarly

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$

$\Rightarrow A_{21} = -(1 - 6) = 5$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$

$\Rightarrow A_{22} = 2 + 21 = 23$

$A_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23}$

$\Rightarrow A_{23} = -(4 + 7) = -11$

And

$A_{31} = (-1)^{3+1} M_{31} = (-1)^4 M_{31}$

$\Rightarrow A_{31} = 0 + 3 = 3$

$A_{32} = (-1)^{3+2} M_{32} = (-1)^5 M_{32}$

$\Rightarrow A_{32} = -(0 - 12) = 12$

$A_{33} = (-1)^{3+3} M_{33} = (-1)^6 M_{33}$

$\Rightarrow A_{33} = -2 - 4 = -6$

Cofactor matrix is $\begin{bmatrix} -1 & -4 & 1 \\ 5 & 23 & -11 \\ 3 & 12 & -6 \end{bmatrix}$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$\Rightarrow \text{adj } A = \begin{bmatrix} -1 & -4 & 1 \\ 5 & 23 & -11 \\ 3 & 12 & -6 \end{bmatrix}^T$

$\therefore \text{adj } A = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$

Inverse of the matrix A is given by,

$A^{-1} = \dfrac{1}{|A|} \text{adj } A$

$\therefore A^{-1} = \dfrac{1}{10} \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$

10.

Find the inverse of each of the matrices (if it exists).

$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

Ans - Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

Expanding along columns C_1,

$|A| = 1(8 - 6) - 0 + 3(3 - 4)$

$\therefore |A| = -1$

Since, Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j}M_{ij}$

$A_{11} = (-1)^{1+1}M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = 8 - 6 = 2$

$A_{12} = (-1)^{1+2}M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = -(0 + 9) = -9$

$A_{13} = (-1)^{1+3}M_{13} = (-1)^4 M_{13}$

$\Rightarrow A_{13} = 0 - 6 = -6$

Similarly

$A_{21} = (-1)^{2+1}M_{21} = (-1)^3 M_{21}$

$\Rightarrow A_{21} = (-4 + 4) = 0$

$A_{22} = (-1)^{2+2}M_{22} = (-1)^4 M_{22}$

$\Rightarrow A_{22} = 4 - 6 = -2$

$A_{23} = (-1)^{2+3}M_{23} = (-1)^5 M_{23}$

$\Rightarrow A_{23} = -(2 + 3) = -1$

And

$A_{31} = (-1)^{3+1}M_{31} = (-1)^4 M_{31}$

$\Rightarrow A_{31} = 3 - 4 = -1$

$A_{32} = (-1)^{3+2}M_{32} = (-1)^5 M_{32}$

$\Rightarrow A_{32} = -(-3 - 0) = 3$

$A_{33} = (-1)^{3+3}M_{33} = (-1)^6 M_{33}$

$\Rightarrow A_{33} = 2 - 0 = 2$

Cofactor matrix is $\begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow \text{adj } A = \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}^T$$

$$\therefore \text{adj } A = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

Inverse of the matrix A is given by,

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\therefore A^{-1} = -1 \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

11.

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$$

Ans - Let A $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$

Expanding along columns C_1,

$|A| = 1(-\cos^2 a - \sin^2 a)$

$\therefore |A| = -1$

Since, Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = -\cos^2 a - \sin^2 a = -1$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = 0$

$A_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$

$\Rightarrow A_{13} = 0$

Similarly

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$

$\Rightarrow A_{21} = 0$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$

$\Rightarrow A_{22} = -\cos a$

$A_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23}$

$\Rightarrow A_{23} = -\sin a$

And

$A_{31} = (-1)^{3+1} M_{31} = (-1)^4 M_{31}$

$\Rightarrow A_{31} = 0$

$A_{32} = (-1)^{3+2} M_{32} = (-1)^5 M_{32}$

$\Rightarrow A_{32} = -\sin a$

$A_{33} = (-1)^{3+3} M_{33} = (-1)^6 M_{33}$

$\Rightarrow A_{33} = \cos a$

Cofactor matrix is $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos a & -\sin a \\ 0 & -\sin a & \cos a \end{bmatrix}$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow adj\ A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos a & -\sin a \\ 0 & -\sin a & \cos a \end{bmatrix}^T$$

$$\therefore adj\ A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos a & -\sin a \\ 0 & -\sin a & \cos a \end{bmatrix}$$

Inverse of the matrix A is given by,

$$A^{-1} = \frac{1}{|A|} adj\ A$$

$$\therefore A^{-1} = -1 \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos a & -\sin a \\ 0 & -\sin a & \cos a \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$$

12.

Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^1 = B^{-1}A^{-1}$.

Ans - Let A $= \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow |A| = 15 - 4$

$\therefore |A| = 1$

Since, Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j}M_{ij}$

$A_{11} = (-1)^{1+1}M_{11} = (-1)^2 M_{11}$

$\Rightarrow A_{11} = 5$

$A_{12} = (-1)^{1+2}M_{12} = (-1)^3 M_{12}$

$\Rightarrow A_{12} = -2$

$A_{21} = (-1)^{2+1}M_{21} = (-1)^{2+1}M_{21}$

$\Rightarrow A_{21} = -7$

$A_{22} = (-1)^{2+2}M_{22} = (-1)^{2+2}M_{22}$

$\Rightarrow A_{22} = 3$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\therefore \text{adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}^T$$

$$\Rightarrow \text{adj } A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

Inverse of the matrix is given by,

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

For $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$$\Rightarrow |B| = 54 - 56$$

$$\therefore |B| = -2$$

$$A_{11} = (-1)^{1+1}M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = 9$$

$$A_{12} = (-1)^{1+2}M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = -7$$

$$A_{21} = (-1)^{2+1}M_{21} = (-1)^{2+1}M_{21}$$

$$\Rightarrow A_{21} = -8$$

$$A_{22} = (-1)^{2+2}M_{22} = (-1)^{2+2}M_{22}$$

$$\Rightarrow A_{22} = 6$$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\therefore \text{adj } A = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}^T$$

$$\Rightarrow \text{adj } A = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

Hence, adj $B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

$$\therefore B^{-1} = \frac{1}{|B|}\text{adj } B = -\frac{1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$\Rightarrow B^{-1} = \begin{bmatrix} \dfrac{9}{2} & 4 \\ \dfrac{7}{2} & -3 \end{bmatrix}$$

Now, multiplying B^{-1} and A^{-1}, we get:

$$B^{-1}A^{-1} = \begin{bmatrix} \dfrac{9}{2} & 4 \\ \dfrac{7}{2} & -3 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\Rightarrow B^{-1}A^{-1} = \begin{bmatrix} -\dfrac{45}{2} - 8 & \dfrac{63}{2} + 12 \\ \dfrac{35}{2} + 6 & -\dfrac{46}{2} - 9 \end{bmatrix}$$

$$\therefore B^{-1}A^{-1} = \begin{bmatrix} -\dfrac{61}{2} & \dfrac{87}{2} \\ \dfrac{47}{2} & -\dfrac{67}{2} \end{bmatrix} \quad \ldots \ldots (1)$$

Similarly, multiplying the matrices A and B, we get:

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 18 + 49 & 24 + 63 \\ 12 + 35 & 16 + 45 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

The value of |AB| is

$$|AB| = 67 \times 61 - 68 \times 47$$

$$\Rightarrow |AB| = 4087 - 4089$$

$$\therefore |AB| = -2$$

The adjoint of (AB) is given by,

$$\text{adj (AB)} = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} \text{adj (AB)}$$

$$\Rightarrow (AB)^{-1} = -\frac{1}{2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$\therefore (AB)^{-1} = \begin{bmatrix} -\dfrac{61}{2} & \dfrac{87}{2} \\ \dfrac{47}{2} & -\dfrac{67}{2} \end{bmatrix} \quad \dots\dots\dots\dots\dots (2)$$

From (1) and (2), we have:

$$(AB)^{-1} = B^{-1}A^{-1}$$

13.

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}.

Ans - Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 = A.A$

$\Rightarrow A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\therefore$ The value of $A^2 - 5A + 7I$ is:

$\Rightarrow A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^2 - 5A + 7I = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\therefore A^2 - 5A + 7I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence, $A^2 - 5A + 7I = 0$

$\Rightarrow A^2 - 5A = -7I$

Multiplying by A^{-1} on both sides, we have:

$\Rightarrow AA(A^{-1}) - 5AA^{-1} = -7IA^{-1}$

$\Rightarrow A(AA^{-1}) - 5I = -7IA^{-1}$

$\Rightarrow AI - 5I = -7IA^{-1}$

$\Rightarrow A^{-1} = -\dfrac{1}{7}(A - 5I)$

$\Rightarrow A^{-1} = \dfrac{1}{7}(5I - A)$

$\Rightarrow A^{-1} = \dfrac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$

$\therefore A^{-1} = \dfrac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

14.

For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, **find the number a and b such that** $A^2 + aA + bI = 0$

Ans - Given $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$A^2 = A.A$

$\Rightarrow A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

Solving $A^2 + aA + bI = 0$ by multiplying the whole equation by A^{-1}.

$\Rightarrow (AA)A^{-1} + aAA^{-1} + bIA^{-1} = 0$

$\Rightarrow A(AA^{-1}) + aI + b(IA^{-1}) = 0$

$\Rightarrow AI + aI + bA^{-1} = 0$

$\Rightarrow A + aI = -bA^{-1}$

$\Rightarrow A^{-1} = \dfrac{1}{b}(A + aI) \ldots \ldots \ldots \ldots \ldots (1)$

To find the value of A^{-1}. We know that the adjoint of a square matrix is the transpose of its cofactor matrix.

$$\Rightarrow adj\ A = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

Inverse is given by, $A^{-1} = \dfrac{1}{|A|} adj\ A.$

$$\Rightarrow A^{-1} = \dfrac{1}{1}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

Substituting values in (1) we get,

$$\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = -\dfrac{1}{b}\left(\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}\right)$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = -\dfrac{1}{b}\begin{bmatrix} 3+a & 2 \\ 1 & 1+a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \dfrac{-3-a}{b} & -\dfrac{2}{b} \\ -\dfrac{1}{b} & \dfrac{-1-a}{b} \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we get:

$$\Rightarrow -\dfrac{1}{b} = -1 \quad \therefore b = 1$$

$$\Rightarrow \dfrac{-3-a}{b} = 1 \quad \therefore a = -4$$

Hence, -4 and 1 are the required values of a and b respectively.

15.

For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 3 \end{bmatrix}$ show that $A^3 - 6A^2 + 5A +$

$11I = 0$. Hence, A^{-1}.

Ans - Given, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 3 \end{bmatrix}$

$A^2 = A.A$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 3 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1+1+1 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$

$A^3 = A^2.A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 3 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -324-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$

Substituting the values for A^3, A^2 and A in $A^3 - 6A^2 + 5A + 11I$, we get,

$A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6 \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$+ 5 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 3 \end{bmatrix} + 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix}$$

$$+ \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 2 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$\Rightarrow A^3 - 6A^2 + 5A + 11I = \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix}$

$$= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 5A + 11I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

Thus, $A^3 - 6A^2 + 5A + 11I = 0$

Multiplying the whole equation by A^{-1}, we have:

$\Rightarrow (AAA)A^{-1} - 6(AA)A^{-1} + 5AA^{-1} + 11IA^{-1} = 0$

$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 5(AA^{-1}) = 11(IA^{-1})$

$\Rightarrow A^2 - 6A + 5I = -11A^{-1}$

$\Rightarrow A^{-1} = -\dfrac{1}{11}(A^2 - 6A + 5I) \dots\dots (1)$

Now, $A^2 - 6A + 5I$ is given by:

$$A^2 - 6A + 5I = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 3 \end{bmatrix}$$
$$+ 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow A^2 - 6A + 5I$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & 6 & 18 \end{bmatrix}$$
$$+ \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow A^2 - 6A + 5I = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 13 & -14 \\ 7 & -3 & 19 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & 6 & 18 \end{bmatrix}$$

$$\therefore A^2 - 6A + 5I = \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

Substituting for $A^2 - 6A + 5I$ equation (1), we get

$$\Rightarrow A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

16.

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ verify that $A^3 - 6A^2 + 9A + 4I = 0$ and hence find A^{-1}.

Ans - Given, $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$A^2 = A.A$

$\Rightarrow A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$

Similarly,

$A^3 = A^2 A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5+10-6 & 5+5+12 \end{bmatrix}$

$\therefore A^3 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$

Now, $A^3 - 6A^2 + 9A + 4I$ is given by:

$A^3 - 6A^2 + 9A + 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$+ 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow A^3 - 6A^2 + 9A + 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix}$$

$$+ \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$\Rightarrow A^3 - 6A^2 + 9A + 4I$

$$= \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} - \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 9A + 4I = 0$

Since, $A^3 - 6A^2 + 9A + 4I = 0$

Multiplying the whole equation by A^{-1}, we have:

$(AAA)A^{-1} - 6(AA)A^{-1} + 9AA^{-1} - 4IA^{-1} = 0$

$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 9(AA^{-1}) = 4(IA^{-1})$

$\Rightarrow AAI - 6AI + 9I = 4A^{-1}$

$\Rightarrow A^2 - 6A + 9I = 4A^{-1}$

$\Rightarrow A^{-1} = \dfrac{1}{4}(A^2 - 6A + 9I) \ldots \ldots \ldots \ldots \ldots (1)$

Now, $A^2 - 6A + 9I$ is given by:

$$A^2 - 6A + 9I = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$
$$+ 9\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^2 - 6A + 9I = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix}$$
$$+ \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$A^2 - 6A + 9I = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 3 & 3 \end{bmatrix}$$

Substituting for $A^2 - 6A + 9I$ equation (1), we get

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 3 & 3 \end{bmatrix}$$

17.

Let A be nonsingular square matrix of order 3 × 3 . Then |adj A| is equal to

A. |A| **B. |A|²**   **C. |A|³**   **D. 3|A|**

Ans - Given A is a non-singular matrix, i.e., it is a square matrix whose determinant is not equal to zero.

Inverse of a matrix is given as $A^{-1} = \frac{1}{|A|} adj\ A$.

$\Rightarrow A^{-1} = \frac{1}{|A|} adj\ A$

$\Rightarrow |A|I = adj\ A$

The adjoint of matrix is given by,

$\Rightarrow (adj\ A) = A = |A|I = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$

$\Rightarrow |(adj\ A)A| = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$

$\Rightarrow |adj\ A||A| = |A|^3 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = |A|^3 (I)$

$\therefore |adj\ A| = |A|^3$

Hence, B. $|A|^2$ is the correct answer.

18.

If A is an invertible matrix of order 2 , then det(A-1) is equal to

A. det(A)

B. $\frac{1}{det(A)}$

C. 1

D. 0

Ans - Since A is a invertible matrix, thus A^{-1} exists and it is given by: $A^{-1} = \frac{1}{|A|} \text{adj A}$.

As matrix A is of order 2,

$\therefore$ Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Hence, $|A| = ad - bc$.

The adjoint of A would be,

$\text{adj A} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now, the inverse of the matrix is given by:

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj A}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \dfrac{d}{|A|} & \dfrac{-b}{|A|} \\ \dfrac{-c}{|A|} & \dfrac{a}{|A|} \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \dfrac{d}{|A|} & \dfrac{-b}{|A|} \\ \dfrac{-c}{|A|} & \dfrac{a}{|A|} \end{bmatrix}$$

$$\Rightarrow |A^{-1}| = \frac{1}{|A^2|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\Rightarrow |A^{-1}| = \frac{1}{|A^2|} (ad - bc)$$

$$\Rightarrow |A^{-1}| = \frac{1}{|A^2|} \cdot |A|$$

$$\therefore |A^{-1}| = \frac{1}{|A|}$$

Thus, $\det(A^{-1}) = \frac{1}{\det A}$

Hence, B. $\frac{1}{\det(A)}$ is the correct answer.

Exercise 4.5

1.

Examine the consistency of the system of equations.

$x + 2y = 2$

$2x + 3y = 3$

Ans – Given system of equations can be written in the form of AX = B, where

$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\Rightarrow |A| = 1(3) - 2(2)$

$\Rightarrow |A| = 3 - 4 = -1 \neq 0$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

Thus, given system of equations is consistent.

2.

Examine the consistency of the system of equations.

$2x - y = 5$

$x + y = 4$

Ans – Given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$\Rightarrow |A| = 2(1) - (-1)(1)$

$\Rightarrow |A| = 2 + 1 = 3 \neq 0$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

Thus, given system of equations is consistent.

3.

Examine the consistency of the system of equations.

$x + 3y = 5$

$2x + 6y = 8$

Ans – Given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$\Rightarrow |A| = 1(6) - 3(2) = 6 - 6 = 0$

$\therefore$ A is a singular matrix.

$$(\text{adj } A) = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

$$(\text{adj } A) B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$$

$\Rightarrow$ The solution of the given system of equations does not exists.

Thus, given system of equations is consistent.

4.

Examine the consistency

$x + y + z = 1$

$2x + 3y + 2z = 2$

$ax + ay + 2az = 4$

Ans – Given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$\Rightarrow |A| = 1(6a-2a) -1(4a-2a)+1(2a-3a)$

$\Rightarrow |A| = 4a - 2a - a$

$\Rightarrow |A| = 4a-3a = a \neq 0$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

Thus, given system of equations is consistent.

5.

Examine the consistency of the system of equations.

$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

Ans – Given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$\Rightarrow |A| = 3(-5) - 0 + 3(1 + 4) = -15 + 15 = 0$

$\therefore$ A is a singular matrix.

$$\text{Now (adj } A) = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$(\text{adj } A)B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10 - 10 + 15 \\ -6 - 6 + 9 \\ -21 - 12 + 18 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$$

$\Rightarrow$ The solution of the given system of equations does not exists.

Thus, given system of equations is consistent.

6.

Examine the consistency of the system of equations.

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

Ans – Given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 3 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$\Rightarrow |A| = 5(18+10) + 1(12-25) + 4(-4-15)$

$\Rightarrow |A| = 5(28) + 1(-13) + 4(-19)$

$\Rightarrow |A| = 140 - 13 - 76 = 51 \neq 0$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

Thus, given system of equations is consistent.

7.

 Solve the system of linear equations, using the matrix method.

$5x + 2y = 4$

$7x + 3y = 5$

Ans - Given system of equation can be written in the form of $AX = B$, where

$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow |A| = 15 - 14 = 1 \neq 0$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

$A^{-1} = \dfrac{1}{|A|}$ (adj A)

$\therefore A^{-1} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

$\therefore X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Hence, $x = 2$ and $y = -3$

8.

Solve the system of linear equations, using the matrix method.

$2x - y = -2$

$3x + 4y = 3$

Ans - Given system of equations can be written in the form of $AX = B$, where

$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$|A| = 8 + 3 = 11 \neq 0$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

$A^{-1} = |A|^{1}(\text{adj } A) = \dfrac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$

$\therefore X = A^{-1}B = \dfrac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{11}\begin{bmatrix} -8 + 3 \\ 6 + 6 \end{bmatrix} = \dfrac{1}{11}\begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} -\dfrac{5}{11} \\ \dfrac{12}{11} \end{bmatrix}$

Hence, $x = -\dfrac{5}{11}$ and $y = \dfrac{12}{11}$.

9.

Solve the system of linear equations, using the matrix method.

$4x - 3y = 3$

$3x - 5y = 7$

Ans - Given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$|A| = -20 + 9 = -11 \neq 0$$

∴ A is non-singular.

⇒ A^{-1} exists.

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = -\frac{1}{11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 15 - 21 \\ 9 - 28 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} -6 \\ -19 \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{6}{11} \\ -\dfrac{19}{11} \end{bmatrix}$$

Hence, $x = -\dfrac{6}{11}$ and $y = -\dfrac{19}{11}$

10.

Solve the system of linear equations, using the matrix method.

$5x + 2y = 3$

$3x + 2y = 5$

Ans - Given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \& B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = 5(2) - 3(2) = 10 - 6 = 4 \neq 0$$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2(3) + (-2)5 \\ -3(3) + 5(5) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 6 - 10 \\ -9 + 25 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -4 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Hence, $x = -1 \& y = 4$

11.

Solve the system of linear equations, using the matrix method.

$2x + y + z = 1$

$x - 2y - z = 32$

$3y - 5z = 9$

Ans - Given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -4 & -2 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$$

$|A| = 2(20 + 6) - 1(-10 - 0) + 1(6 - 0)$

$= 52 + 10 + 6 = 68 \neq 0$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

Hence, the given system is consistent and has a unique solution given by $X = A^{-1}B$.

Cofactor of A are.

$A_{11} = 20 + 6 = 26$

$A_{12} = -(-10 + 0) = 10$

$A_{13} = 6 + 0 = 6$

$A_{21} = -(-5 - 3) = 8$

$A_{22} = -10 - 0 = -10$

$A_{23} = -(6 - 0) = -6$

$A_{31} = (-2 + 4) = 2$

$A_{32} = -(-4 - 2) = 6$

$A_{33} = -8 - 2 = -10$

$$\text{adj } (A) = \begin{bmatrix} 26 & 10 & 6 \\ 8 & -10 & -6 \\ 2 & 6 & -10 \end{bmatrix}^T$$

$$= \begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{68} \begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$= \frac{1}{68} \begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$$

$$= \frac{1}{68} \begin{bmatrix} 68 \\ 34 \\ -102 \end{bmatrix} = \begin{bmatrix} 1 \\ \dfrac{1}{2} \\ \dfrac{-3}{2} \end{bmatrix}$$

Hence, $x = 1, y = \dfrac{1}{2}$ and $z = \dfrac{-3}{2}$

12.

Solve a system of linear equations, using matrix method.
$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

Ans - Given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$|A| = 1(1 + 3) + 1(2 + 3) + 1(2 - 1) = 4 + 5 + 1 = 10 \neq 0$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

$A_{11} = 4, A_{12} = -5, A_{13} = 1$

$A_{31} = 2, A_{22} = 0, A_{23} = -2$

$A_{31} = 2, A_{32} = 5, A_{33} = 3$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 + 0 + 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Hence, $x = 2, y = -1, \& z = 1$

13.

Solve the system of linear equations, using the matrix method.

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

Ans - Given system of equations can be written in the form of AX = B, where

$|A| = 2(4 + 1) - 3(2 - 3) + 3(-1 + 6)$

$= 2(5) - 3(-5) + 3(5)$

$= 10 + 15 + 15 = 40 \neq 0$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

$A_{11} = 5, A_{12} = 5, A_{13} = 5$

$A_{21} = 3, A_{22} = -13, A_{23} = -11$

$A_{31} = 9, A_{32} = 1, A_{33} = -7$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 22 + 3 \\ 25 - 44 - 21 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x = 1, y = 2$ and $z = -1$

14.

Solve the system of linear equations, using the matrix method.

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

Ans - Given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -11 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$|A| = 1(12 - 5) + 1(9 + 10) + 2(-3 - 8) = 7 + 19 - 22 = 4 \neq 0$

$\therefore$ A is non-singular.

$\Rightarrow A^{-1}$ exists.

$A_{11} = 7, A_{12} = -19, A_{13} = 11$

$A_{21} = 1, A_{22} = -1, A_{23} = -1$

$A_{31} = -3, A_{32} = 11, A_{33} = 7$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 49 - 5 - 36 \\ -133 + 5 + 132 \\ -77 + 5 + 84 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $x = 2, y = 1$ and $z = 3$

15.

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} using A^{-1} solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

Ans - $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$\therefore |A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = 0 - 6 + 5$
$$= -1 \neq 0$$

$A_{11} = 0, A_2 = 2, A_3 = 1$

$A_{21} = -1, A_{22} = -9, A_{23} = -5$

$A_{31} = 2, A_{32} = 23, A_{33} = 13$

$\therefore A^{-1} = \dfrac{1}{|A|} (\text{adj } A) = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Given system of equation can be written in the form of $AX = B$, where

$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

The solution o the system of equation is given by

$\therefore X = A^{-1}B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Hence, $x = 1, y = 2$ and $z = 3$

16.

The cost of 4Kg onion, 3kg wheat and 2kg rice is Rs60. The cost of 2kg onion, 4kg wheat and 6Kg rice is Rs 90. The cost of 6kg onion 2kg wheat and 3kg rice is Rs 70 .

Find cost of each item per kg by matrix method

Ans - Given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$|A| = 4(12-12) - 3(6-36) + 2(4-24)$

$|A| = 0 + 90 - 40 = 50 \neq 0$

$A_{11} = 0, A_2 = 30, A_3 = -20$

$A_{21} = -5, A_{22} = 0, A_{23} = 10$

$A_{31} = 10, A_{32} = -20, A_{33} = 10$

$$\therefore \text{adj } A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\therefore A^{-1} = A|^1 \text{ adj } A = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow X = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 + 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore x = 5, y = 8,$ and $z = 8$

Thus, the cost of onions is Rs 5 per kg, the cost of wheat is Rs 8 per kg, and the cost of rice is Rs 8 per kg.