NCERT Solutions for Class 12 Maths Chapter 3 – Matrices

Exercise 3.1

1.

In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

(i) The order of the matrix,
(ii) The number of elements,
(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Ans – (i) Order of the given matrix A is 3 x 4
(ii) Number of elements will be 3 * 4 = 12
(iii) $a_{13} = 19$, $a_{21} = 35$, $a_{33} = -5$, $a_{24} = 12$, $a_{23} = 5/2$

2.

If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Ans – In order to get the possible orders of matrix with 24 elements we need to find all the pair of natural numbers whose product will be 24. They are (1 × 24), (24 × 1), (2 × 12), (12 × 2), (3 × 8), (8 × 3), (4 × 6), (6 × 4).
Similarly for 13 elements is will be (13 x 1) and (1 x 13).

3.

If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Ans – In order to get the possible orders of matrix with 18 elements we need to find all the pair of natural numbers whose product will be 18. They are (1 × 18), (18 × 1), (2 × 9), (9 × 2),

(3×6), (6×3).

Similarly for 5 elements is will be (5 x 1) and (1 x 5).

4.

Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$

(ii) $a_{ij} = \frac{i}{j}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Ans - (i) $a_{ij} = \frac{(i+j)^2}{2}$

Elements for 2×2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

$$a_{11} = \frac{(1+1)^2}{2} = \frac{(2)^2}{2} = 2$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = 8$$

So, the required matrix is: $\begin{pmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{pmatrix}$

(ii) $a_{ij} = \frac{i}{j}$

Elements for 2×2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

$$a_{11} = \frac{1}{1} = 1$$

$$a_{12} = \frac{1}{2}$$

$$a_{21} = \frac{2}{1} = 2$$

$$a_{22} = \frac{2}{2} = 1$$

So, the required matrix is: $\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{pmatrix}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Elements for 2×2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

$$a_{11} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

$$a_{12} = \frac{(1+4)^2}{2} = \frac{(5)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = 8$$

$$a_{22} = \frac{(2+4)^2}{2} = \frac{(6)^2}{2} = 18$$

So, the required matrix is: $\begin{pmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{pmatrix}$

5.

Construct a 3×4 matrix, whose elements are given by:

(i) $\boldsymbol{a_{ij} = \frac{1}{2}|-3i + j|}$

(ii) $\boldsymbol{a_{ij} = 2i - j}$

Ans - **(i)** Given that $a_{ij} = \frac{1}{2}|-3i + j|$

$$\therefore\ a_{11} = \frac{1}{2}|-3 \times 1 + 1| = 1$$

$$a_{21} = \frac{1}{2}|-3 \times 2 + 1| = \frac{5}{2}$$

$$a_{31} = \frac{1}{2}|-3 \times 2 + 1| = 4$$

$$a_{12} = \frac{1}{2}|-3 \times 1 + 2| = \frac{1}{2}$$

$$a_{22} = \frac{1}{2}|-3 \times 2 + 2| = 2$$

$$a_{32} = \frac{1}{2}|-3 \times 3 + 2| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}|-3 \times 1 + 3| = 0$$

$$a_{23} = \frac{1}{2}|-3 \times 2 + 3| = \frac{3}{2}$$

$$a_{33} = \frac{1}{2}|-3 \times 3 + 3| = 3$$

$$a_{14} = \frac{1}{2}|-3 \times 1 + 4| = \frac{1}{2}$$

$$a_{24} = \frac{1}{2}|-3 \times 2 + 4| = 1$$

$$a_{34} = \frac{1}{2}|-3 \times 3 + 4| = \frac{5}{2}$$

Thus, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

(ii) A 3×4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

Given that $a_{ij} = 2i - j$,

$\therefore a_{11} = 2 \times 1 - 1 = 1$

$a_{21} = 2 \times 2 - 1 = 3$

$a_{31} = 2 \times 3 - 1 = 5$

$a_{12} = 2 \times 1 - 2 = 0$

$a_{22} = 2 \times 2 - 2 = 4$

$a_{32} = 2 \times 3 - 2 = 4$

$a_{13} = 2 \times 1 - 3 = -1$

$a_{23} = 2 \times 2 - 3 = 1$

$a_{33} = 2 \times 3 - 3 = 3$

$a_{14} = 2 \times 1 - 4 = -2$

$a_{24} = 2 \times 2 - 4 = 0$

$a_{34} = 2 \times 3 - 4 = 2$

Thus, the required matrix is $A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$.

6.

Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Ans - **(i)** Given $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

Comparing the corresponding elements, we get,

$x = 1, y = 4, z = 3$

(ii) Given $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Comparing the corresponding elements, we get,

$x + y = 6, xy = 8, 5 + z = 5$

Now, $\because 5 + z = 5$

$\Rightarrow z = 0$

We know that, $(x - y)^2 = (x + y)^2 - 4xy$

$\Rightarrow (x - y) = \pm 2$

When $(x - y) = 2$ and $(x + y) = 6$,

We get $x = 4, y = 2$

When $(x - y) = -2$ and $(x + y) = 6$,

We get $x = 2, y = 4$

$\therefore x = 4, y = 2, z = 0$ *or* $\therefore x = 2, y = 4, z = 0$

(iii) Given $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Comparing the corresponding elements, we get,

$x + y + z = 9$ ... (1)

$x + z = 5$ .... (2)

$y + z = 7$ ... (3)

From equation (1) and (2),

$y + 5 = 9$

$\Rightarrow y = 4$

From equaton (3) we have,

$4 + z = 7$

$\Rightarrow z = 3$

$x + z = 5$

$\Rightarrow x = 2$

$\therefore x = 2, y = 4, z = 3$

7.

Find the value of a, b, c and d from the equation:

$$\begin{bmatrix} \boldsymbol{a-b} & \boldsymbol{2a+c} \\ \boldsymbol{2a-b} & \boldsymbol{3c+d} \end{bmatrix} = \begin{bmatrix} \mathbf{-1} & \mathbf{5} \\ \mathbf{0} & \mathbf{13} \end{bmatrix}$$

Ans - Given $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Comparing the corresponding elements, we get,

$a - b = -1$ ...(1)

$2a - b = 0$ ...(2)

$2a + c = 5$ ...(3)

$3c + d = 13$ ... (4)

From equation (2),

$b = 2a$

From equation (1),

$a - 2a = -1$

$\Rightarrow a = 1$

$\Rightarrow b = 2$

From equaiton (3),

$2 \times 1 + c = 5$

$\Rightarrow c = 3$

From equation (4),

$3 \times 3 + d = 13$

$\Rightarrow d - 4$

$\therefore a = 1, b = 2, c = 3, d = 4$

8.

$A = [a_y]_{m\times n}$ is a square matrix, if

A. $m < n$ B. $m > n$ C. $m = n$ D. None of these

Ans - A given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

$\therefore A = [a_y]_{m\times n}$ is a square matrix if, $m = n$

Thus, option (C) is correct.

9.

Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

A. $x = \frac{11}{3}, y = 7$

B. Not possible to find

C. $y = 7, x = \frac{-2}{3}$

D. $x = \frac{-1}{3}, y = \frac{-2}{3}$

Ans - Given $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Comparing the corresponding elements, we get,

$3x + 7 = 0$

$\Rightarrow x = -\frac{7}{3}$

$y - 2 = 5$

$\Rightarrow y = 7$

$y + 1 = 8$

$\Rightarrow y = 7$

$2 - 3x = 4$

$\Rightarrow x = -\frac{2}{3}$

10.

The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

(A) 27 (B) 18 (C) 81 (D) 512

Ans – Given 3 x 3 matrix means it will have 9 elements. Each element can be filled in two possible ways i.e. 0 or 1. So the number of all possible matrices will be 2^9 = 512. Hence option D is the correct answer.

Exercise 3.2

1.

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Find each of the following:

(i) $A + B$ **(ii)** $A - B$ **(iii)** $3A - C$

(iv) AB **(v)** BA

Ans - (i) Adding the matrices

$$A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

(ii) Subtracting the matrices

$$A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

(iii) Subtracting the matrices

$$3A - C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

(iv) Matrix A has 2 columns. This number is equal to the number of rows in matrix B. Therefore, AB is defined as:

$$AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2(1) + 4(-2) & 2(3) + 4(5) \\ 3(1) + 2(-2) & 3(3) + 2(5) \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ 1 & 19 \end{bmatrix}$$

2.

Compute the following:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

(iv) $\begin{bmatrix} cos^2x & sin^2x \\ sin^2x & cos^2x \end{bmatrix} + \begin{bmatrix} sin^2x & cos^2x \\ cos^2x & sin^2x \end{bmatrix}$

Ans - **(i)** Computing,

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

(ii) Computing,

$$\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} a^2+b^2+2ab & b^2+c^2-2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$\begin{bmatrix} a^2+b^2+2ab & b^2+c^2-2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

(iii) Computing,

$$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

(iv) Computing,

$$\begin{bmatrix} cos^2x & sin^2x \\ sin^2x & cos^2x \end{bmatrix} + \begin{bmatrix} sin^2x & cos^2x \\ cos^2x & sin^2x \end{bmatrix} = \begin{bmatrix} cos^2x+sin^2x & sin^2x+cos^2x \\ sin^2x+cos^2x & cos^2x+sin^2x \end{bmatrix}$$

$$\begin{bmatrix} cos^2x+sin^2x & sin^2x+cos^2x \\ sin^2x+cos^2x & cos^2x+sin^2x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, (sin^2x+cos^2x = 1)$$

3.

Compute the indicated products.

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ **(iv)** $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$ **(vi)** $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

Ans - **(i)** Given $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a(a) + b(b) & a(-b) + a(b) \\ -b(a) + b(a) & -b(-b) + a(a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$$

(ii) Given $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 230 & 2(4) \\ 3(2) & 3(3) & 3(4) \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

(iii) Given: $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1) - 2(2) & 1(2) - 2(3) & 1(3) - 2(1) \\ 2(1) + 3(2) & 2(2) + 3(3) & 2(3) + 3(1) \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

(iv) Given: $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix}\begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

$$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix}\begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5) \\ 4(1)+5(0)+6(3) & 4(3)+5(2)+6(0) & 4(5)+5(4)+6(5) \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

(v) Given $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

(vi) Given $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

$$\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1) \end{bmatrix}$$
$$= \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

4.

If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & 5 \\ 2 & -2 & 3 \end{bmatrix}$ $and\ C =$ $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$. then compute $(A + B) and\ (B - C)$. Also verify that $A + (B - C) = (A + B) - C$

Ans –

$$(A + B) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & 5 \\ 2 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

$$(B - C) = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & 5 \\ 2 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

To verify $A + (B - C) = (A + B) - C$, we calculate $A + (B - C)$ and $(A + B) - C$ first, and then check that LHS = RHS

Calculating $A + (B - C)$

$$\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix} \text{ and}$$

Calculating $(A + B) - C$

$$\begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

$\therefore$ we have verified that $A + (B - C) = (A + B) - C$.

5.

If $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$ then calculate $3A - 5$.

Ans - Evaluating $3A - 5B$, $\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix} =$

$\begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

6.

Simplify $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

Ans - Simplifying $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} +$
$\sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $(\because \cos^2\theta + \sin^2\theta = 1)$

7.

Find X and Y, if

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ **and** $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ **and** $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Ans - **(i)** Given:

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad ...(i)$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad ...(ii)$$

Adding these two equations, we get

$$2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Putting X in (1)

$$\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} Y = \begin{bmatrix} 7-5 & 0-0 \\ 2-1 & 5-4 \end{bmatrix}$$

$$Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

(ii) Given:

$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ ... (i)

$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ .....

Multiplying (1) and (2), we have $4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$ ... (3)

Multiplying (2) with 3, we have

$9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$ ... (4)

From (3) and (4), we have

$$-5X = \begin{bmatrix} 4-6 & 6-(-6) \\ 8-(-3) & 0-15 \end{bmatrix} \therefore X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

Putting X in (1)

$$2\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} 3Y = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

8.

Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Ans - Given:

$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}, where\ Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} 2X = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$

$\therefore X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

9.

Find x and y, if $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

Ans - Given: $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix} \Rightarrow \begin{bmatrix}2+y & 6+0\\0+1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

Comparing the corresponding elements of these two matrices, we have:

$2+y=5 \Rightarrow y=3$

$2x+2=8 \Rightarrow x=3$

$\therefore x=3 \text{ and } y=3$

10.

Solve the equation for x, y, z and t if:
$2\begin{bmatrix}x & z\\y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\4 & 6\end{bmatrix}$

Ans - Given:

$2\begin{bmatrix}x & z\\y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\4 & 6\end{bmatrix}$

$\begin{bmatrix}2x & 2z\\2y & 2t\end{bmatrix}+\begin{bmatrix}3 & -3\\0 & 6\end{bmatrix}=\begin{bmatrix}9 & 15\\12 & 18\end{bmatrix}$

$\begin{bmatrix}2x+3 & 2z-3\\2y & 2t+6\end{bmatrix}=\begin{bmatrix}9 & 15\\12 & 18\end{bmatrix}$

11.

If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$, find values of x and y.

Ans - Given:

$x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\begin{bmatrix}2x-y\\3x+y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

12.

Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$**, find the values of** x, y, z **and** W**.**

13.

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$**, show that** $F(x)F(y) = F(x+y)$**.**

Ans - To show $F(x)F(y) = F(x+y)$,

We first calculate $F(x)F(y)$ *and* $F(x+y)$, and check that both are equal LHS

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x)F(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x)F(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

RHS

$$F(x)F(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

LHS = RHS proved.

14.

Show that

(i) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Ans – **(i)** To Show, we first calculate LHS and RHS

LHS

$$\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix} = \begin{bmatrix}5(2)-1(3) & 5(1)-1(4)\\6(2)+7(3) & 6(1)+7(4)\end{bmatrix}$$
$$= \begin{bmatrix}10-3 & 5-4\\12+21 & 6+28\end{bmatrix} = \begin{bmatrix}7 & 1\\33 & 34\end{bmatrix}$$

RHS

$$\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix} = \begin{bmatrix}2(5)-1(6) & 2(-1)-1(7)\\3(5)+4(6) & 3(-1)+4(7)\end{bmatrix}$$
$$= \begin{bmatrix}10+6 & -2+7\\15+24 & -3+28\end{bmatrix} = \begin{bmatrix}16 & 5\\39 & 25\end{bmatrix}$$

$\therefore LHS \neq RHS$

(ii) To show, we first calculate LHS and RHS

LHS

$$\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}$$
$$= \begin{bmatrix}1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4)\\0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4)\\1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4)\end{bmatrix}$$

$$= \begin{bmatrix}5 & 8 & 14\\0 & -1 & 1\\-1 & 0 & 1\end{bmatrix}$$

RHS

$$\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$$
$$= \begin{bmatrix}-1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0)\\0(1)-1(0)+1(1) & 0(2)-1(1)+1(1) & 0(3)-1(0)+1(0)\\2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0)\end{bmatrix}$$

$$= \begin{bmatrix}-1 & -1 & 3\\6 & 0 & 0\\6 & 11 & 6\end{bmatrix}$$

$\therefore LHS \neq RHS$

15.

Find $A^2 - 5A + 6I$ if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Ans - We know that, $A^2 = A \times A$

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+3(-1) & 2(1)+1(3)+3(0) \\ 1(2)-1(2)+0(1) & 1(0)-1(1)+0(-1) & 1(1)-1(3)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 1 & -1 & -2 \end{bmatrix}$$

Substituting Value of $A^2, A, I \; in \; A^2 - 5A + 6I$

$$\Rightarrow \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 1 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5-10+6 & -1 & 2-5 \\ 9-10 & -2-5+6 & 5-15 \\ -5+0 & -1+5 & -2-0+6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

16.

If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, Prove that $A^3 - 6A^2 + 7A + 2I = 0$

Ans - As we know that, $A^2 = A \times A$ and $A^3 = A^2 \times A$

$$A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+0(0)+2(2) & 1(0)+0(2)+2(0) & 1(2)+0(1)+2(3) \\ 0(1)+2(0)+1(2) & 0(0)+2(2)+1(0) & 0(2)+2(1)+1(3) \\ 2(1)+0(0)+3(2) & 2(0)+0(2)+3(0) & 2(2)+0(1)+3(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+2 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5(1)+0(0)+8(2) & 5(0)+0(2)+8(0) & 5(2)+0(1)+8(3) \\ 2(1)+4(0)+5(2) & 2(0)+4(3)+5(0) & 2(2)+4(1)+5(3) \\ 8(1)+0(0)+13(2) & 8(0)+0(2)+13(0) & 8(2)+0(1)+13(3) \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Substituting value of A^3, A^2, A, I in $A^3 - 6A^2 + 7A + 2I = 0$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + 2\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 7A + 2I = 0$

17.

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

Ans - We know that, $A^2 = A \times A$

$$\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3(3) - 2(4) & 3(-2) - 2(-2) \\ 4(3) - 2(4) & 4(-2) - 2(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now, $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k - 2 & -2k \\ 4k & -2k - 2 \end{bmatrix}$$

Comparing the corresponding elements, we have:

$\Rightarrow 3k - 2 = 1$

$\Rightarrow k = 1$

18.

If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Ans - LHS

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

RHS

$$(I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\, tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\, tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \left(2\cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} \\ -(2\cos^2\frac{\alpha}{2} - 1)\tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin^2\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

$\Rightarrow$ LHS = RHS.

19.

A trust fund has Rs30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of :

(a) Rs 1,800 (b) Rs 2,000

$$\Rightarrow [x \quad 30{,}000]\begin{bmatrix}\frac{5}{100}\\ \frac{7}{100}\end{bmatrix} = 1{,}800$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30{,}000 - x)}{100} = 1{,}800$$

$$\Rightarrow 5x + 2{,}10{,}000 - 7x = 1{,}80{,}000$$

$$\Rightarrow 2x = 30,000$$

$$\Rightarrow x = 15,000$$

$$\Rightarrow [x \quad 30{,}000]\begin{bmatrix}\frac{5}{100}\\ \frac{7}{100}\end{bmatrix} = 2,000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30{,}000 - x)}{100} = 20,000$$

$$\Rightarrow 5x + 2{,}10{,}000 - 7x = 2{,}00{,}000$$

$$\Rightarrow 2x = 10,000$$

$$\Rightarrow x = 5,000$$

20.

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs 80, Rs 60 and Rs 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

$$\Rightarrow 12[10 \quad 8 \quad 10]\begin{bmatrix}80\\60\\40\end{bmatrix}$$

$$\Rightarrow 12[10 \times 80 + 8 \times 60 + 10 \times 40]$$

$$\Rightarrow 12(800 + 480 + 400)$$

$$\Rightarrow 12(1680)$$

$$\Rightarrow 20160$$

21.

Assume X, Y, Z, W and P are the matrices of order $2\times n$, $3\times k$, $2\times p$, $n\times 3$ and $p\times k$ respectively. The restriction on n, k and p so that $PY + WY$ will be defined.

A. $k = 3, p = n$

B. k is arbitrary, $p = 2$

C. p is arbitrary, $k = 2$

D. $k = 2, p = 3$

22.

Assume X, Y, Z, W and P are the matrices of order $2\times n$, $3\times k$, $2\times p$, $n\times 3$ and $p\times k$ respectively. If $n = p$, then the order of the matrix $7X - 5Z$ is

A. $p\times 2$

B. $2\times n$

C. $n\times 3$

D. $p\times n$

Chapter 3 – Matrices Exercise 3.3

1.

Find the transpose of wach of the following matrices:

(i) $\begin{bmatrix} 5 \\ 1/2 \\ -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Ans - (i) $A = \begin{bmatrix} 5 \\ 1/2 \\ -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 2 \\ - & 3 \end{bmatrix}$

(iii) $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

2.

If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$

(ii) $(A - B)' = A' - B'$

Ans - We can get

$$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

(i) $A + B = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix} \Rightarrow (A + B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

$$A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$\Rightarrow (A + B)' = A' + B'$

(ii) $A - B = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix} \Rightarrow (A - B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$

$$A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$\Rightarrow (A - B)' = A' - B'$

3.

If $\boldsymbol{A'} = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ **and** $\boldsymbol{B} = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$**, then verify that**

(i) $(\boldsymbol{A} + \boldsymbol{B})' = \boldsymbol{A'} + \boldsymbol{B'}$ **(ii)** $(\boldsymbol{A} - \boldsymbol{B})' = \boldsymbol{A'} - \boldsymbol{B'}$

Ans - We can get

$$A' = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} \quad B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

(i) $A + B = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix} \Rightarrow (A+B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$

$$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$\Rightarrow (A+B)' = A' + B'$

(ii) $A - B = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix} \Rightarrow (A-B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

$$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$\Rightarrow (A-B)' = A' - B'$

4.

If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$**, then find** $(A + 2B)'$

Ans -

$$A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

Solve for the condition

$$\therefore A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$$\Rightarrow A + 2B = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

5.

For the matrices A and B, verify that $(AB)' = B'A'$, where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Ans - (i) $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

$\Rightarrow (AB)' = B'A'$

(ii) $AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 3 \end{bmatrix}$

$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

$B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

$\Rightarrow (AB)' = B'A'$

6.

If (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$**, then verify that** $A'A = I$

(ii) $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$**, then verify that** $A'A = I$

Ans – **(i)** $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\therefore A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$\Rightarrow A'A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

(ii) $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$\therefore A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$\Rightarrow A'A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

7.

(i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ **is a symmetric matrix.**

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ **is a skew symmetric matrix.**

Ans - (i) we have

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

$\therefore A' = A$

Hence, A is a symmatric matrix.

(ii) We have

$$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

$\therefore A' = -A$

Hence, A is a skew-symmatric matrix.

8.

For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix

(ii) $(A + A')$ is a skew symmetric matrix

Ans - $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}, A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

(i) $A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$(A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$

$\Rightarrow (A + A')$ is a symmetric matrix

(ii) $A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = A + A'$

$\Rightarrow (A - A')$ is a skew symmetric matrix.

9.

Find $\frac{1}{2}(A + A')$ & $\frac{1}{2}(A - A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Ans - Given matrix $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

$$\Rightarrow A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$\frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}\right)$$

$$\frac{1}{2}(A + A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}\right)$$

$$\frac{1}{2}(A - A') = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

10.

Express the following matrices as the sum of symmetric and a skew symmetric matrix.

$$\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

Ans - $\frac{1}{2}(A+A') = \frac{1}{2}\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

$$\frac{1}{2}(A+A') = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\frac{1}{2}(A+A') = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$\Rightarrow \frac{1}{2}(A+A')$ is a symmetric matrix.

$$\frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$\frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A-A') = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$\Rightarrow \frac{1}{2}(A-A')$ is a skew- symmetric matrix.

11.

If A, B are symmetric matrix of some order, then AB – BA is a

A. Skew symmetric matrix

B. Symmetric matrix

C. Zero matrix

D. Identity matrix

Ans - The correct answer is A

A & B are symmetric, therefore we have

$A' = A \& B' = B$

Consider

$(AB - BA)' = (AB)' - (BA)'$

$= B'A' = A'B'$

$= BC - AB$

$= -(AB - BA)$

$\therefore (AB - BA)' = -(AB - BA)$

Thus, $(AB - BA)'$ is a skew-symmetric matrix

12.

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$**, then** $A + A' = I$**, if the value of** α ***is***

A. $\frac{\pi}{6}$ **C.** π

B. $\frac{\pi}{3}$ **D.** $\frac{3\pi}{2}$

Ans - The correct answer is B

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A + A' = I$$

$$\therefore \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha = \frac{\pi}{3}$$

Exercise 3.4

1.

Matrices A and B will be inverse of each other only if

(A) AB = BA **(B) AB = BA = 0**

(C) AB = 0, BA = I **(D) AB = BA = I**

Ans - The correct answer is option (D)

If A is a square matrix of order m and there exists another square matrix B of the same order m where AB=BA=I, then B is said to be the inverse of A. In such a circumstance, A is clearly the inverse of B.

Therefore, matrices A and B will be inverses of one another if and only if AB = BA = I.