

# NCERT Solutions for Class 12 Maths

## Chapter 2 – Inverse Trigonometric Functions

### Exercise 2.1

1.

Find the principle value of  $\sin^{-1}\left(-\frac{1}{2}\right)$

**Ans** - Let  $\sin^{-1}\left(-\frac{1}{2}\right) = y$

$$\Rightarrow \sin y = \left(-\frac{1}{2}\right) = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right).$$

As we know that the range of the principle value branch of  $\sin^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  and  $\sin\left(-\frac{\pi}{6}\right) = \frac{1}{2}$

$\therefore$  the principle value of  $\sin^{-1}\left(-\frac{1}{2}\right)$  is  $-\frac{\pi}{6}$ .

2.

Find the principle value of  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ .

**Ans** - Let  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$ .

$$\text{Then } \cos y = \left(\frac{\sqrt{3}}{2}\right) = \cos\left(\frac{\pi}{6}\right).$$

As we know that the range of the principle value branch of  $\cos^{-1}$  is  $[0, \pi]$  and  $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\therefore$  the principle value of  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$  is  $\frac{\pi}{6}$ .

3.

**Find the principle value of  $\operatorname{cosec}^{-1}(2)$ .**

**Ans** - Let  $\operatorname{cosec}^{-1}(2) = y$ .

Then,  $\operatorname{cosec} y = 2 = \operatorname{cosec}\left(\frac{\pi}{6}\right)$ .

As we know that the range of the principle value branch of  $\operatorname{cosec}^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ .

$\therefore$  the principle value of  $\operatorname{cosec}^{-1}(2)$  is  $\frac{\pi}{6}$ .

4.

**Find the principle value of  $\tan^{-1}(-\sqrt{3})$**

**Ans** - Let  $\tan^{-1}(-\sqrt{3}) = y$ ,

Then,  $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$ .

As we know that the range of the principle value branch of  $\tan^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  and  $\tan\left(-\frac{\pi}{3}\right)$  is  $-\sqrt{3}$ .

$\therefore$  the principle value of  $\tan^{-1}(-\sqrt{3})$  is  $-\frac{\pi}{3}$ .

5.

**Find the principle value of  $\cos^{-1}\left(-\frac{1}{2}\right)$**

**Ans** - Let  $\cos^{-1}\left(-\frac{1}{2}\right) = y$ .

Then,  $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$ .

As we know that the range of the principle value branch of  $\cos^{-1}$  is  $[0, \pi]$  and  $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

$\therefore$  the principle value of  $\cos^{-1}\left(-\frac{1}{2}\right)$  is  $\left(\frac{2\pi}{3}\right)$ .

6.

**Find the value of  $\tan^{-1}(-1)$**

**Ans** - Let  $\tan^{-1}(-1) = y$ .

Then,  $\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$ .

As we know that the range of the principle value branch of  $\tan^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  and  $\tan\left(-\frac{\pi}{4}\right) = -1$ .

$\therefore$  the principle value of  $\tan^{-1}(-1)$  is  $-\frac{\pi}{4}$ .

7.

**Find the principle value of  $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$**

**Ans** - Let  $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$ .

Then  $\sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right)$ .

As we know that the range of the principle value branch of  $\sec^{-1}$  is  $[0, \pi] - \left\{\frac{\pi}{2}\right\}$  and  $\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$ .

8.

**Find the principle value of  $\cot^{-1}\sqrt{3}$ .**

**Ans** - Let  $\cot^{-1}\sqrt{3} = y$ . Then  $\cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$

As we know that the range of the principle value branch of  $\cot^{-1}$  is  $[0, \pi]$  and  $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$ .

$\therefore$  the principle value of  $\cot^{-1}\sqrt{3} = \frac{\pi}{6}$ .

9.

**Find the principle value of  $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$**

**Ans** - Let  $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$ .

$$\text{Then } \cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$$

As we know that the range of the principle value branch of  $\cos^{-1}$  is  $[0, \pi]$  and  $\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$\therefore$  the principle value of  $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$  is  $\frac{3\pi}{4}$ .

10.

**Find the principle value of  $\operatorname{cosec}^{-1}(-\sqrt{2})$**

**Ans** - Let  $\operatorname{cosec}^{-1}(-\sqrt{2}) = y$ .

$$\text{Then, } \operatorname{cosec} y = -\sqrt{2} = -\operatorname{cosec}\left(\frac{\pi}{4}\right) = \operatorname{cosec}\left(-\frac{\pi}{4}\right).$$

As we know that the range of the principle value branch of  $\operatorname{cosec}^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$  and  $\operatorname{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}$ .

$\therefore$  the principle value of  $\operatorname{cosec}^{-1}(-\sqrt{2})$  is  $-\frac{\pi}{4}$ .

11.

**Find the value of  $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$**

**Ans** - Let  $\tan^{-1}(1) = x$ .

Then,  $\tan x = 1 = \tan\left(\frac{\pi}{4}\right)$ .

$$\therefore \tan^{-1}(1) = \frac{\pi}{4}$$

Let us assume,  $\cos^{-1}\left(-\frac{1}{2}\right) = y$

Then,  $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$ .

$$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

Let's again assume that  $\sin^{-1}\left(-\frac{1}{2}\right) = z$

Then,  $\sin z = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

12.

**Find the value of  $\cos^{-1}\left(\frac{1}{2}\right) + 2 \sin^{-1}\left(\frac{1}{2}\right)$**

**Ans** - Let's consider,  $\cos^{-1}\left(\frac{1}{2}\right) = x$

Then,  $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$ .

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Let's assume  $\sin^{-1}\left(\frac{1}{2}\right) = y$ .

Then,  $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$ .

$$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2 \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2 \times \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

13.

**If  $\sin^{-1}x = y$ , then**

(A)  $0 \leq y \leq \pi$

(B)  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(C)  $0 < y < \pi$

(D)  $-\frac{\pi}{2} < y < \frac{\pi}{2}$

**Ans** - It is given that  $\sin^{-1}x = y$ .

As we know that the range of the principle value branch of

$\sin^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

Therefore,  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ .

Hence option (B) is correct.

14.

**$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$  is equal to**

(A)  $\pi$

(B)  $-\pi/3$

(C)  $\pi/3$

(D)  $2\pi/3$

**Ans** - Let's consider,  $\tan^{-1}\sqrt{3} = x$ .

Then,  $\tan x = \sqrt{3} = \tan \frac{\pi}{3}$

As we know that the range of the principle value branch of  $\tan^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \therefore$

$$\tan^{-1}\sqrt{3} = \frac{\pi}{3}.$$

Let assume,  $\sec^{-1}(-2) = y$ .

Then,  $\sec y = -2 = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\frac{2\pi}{3}$ .

As we know that the range of the principle value branch of  $\sec^{-1}$  is  $[0, \pi] - \left\{\frac{\pi}{2}\right\} \therefore$

$$\sec^{-1}(-2) = \frac{2\pi}{3}$$

Thus,  $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

Therefore,  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ .

## Exercise 2.1

1.

**Prove**  $3\sin^{-1}x = \sin^{-1}(3x - 4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

**Ans** - To prove  $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$ , where  $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Let  $x = \sin \theta$ . Then,  $\sin^{-1}x = \theta$ .

We have given that,

$$\text{R.H.S } \sin^{-1}(3x - 4x^3) = \sin^{-1}(3\sin \theta - 4\sin^3 \theta)$$

$$= \sin^{-1}(\sin 3\theta)$$

$$= 3\theta$$

$$= 3\sin^{-1}x = \text{L.H.S}$$

Hence proved.

2.

**Prove**  $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$ ,  $x \in \left[\frac{1}{2}, 1\right]$

**Ans** - To prove  $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$ ,  $x \in \left[\frac{1}{2}, 1\right]$

Let  $x = \cos \theta$ . Then,  $\cos^{-1}x = \theta$

We have given that,

$$\text{R.H.S } = \cos^{-1}(4x^3 - 3x)$$

$$= \cos^{-1}(4\cos^3 \theta - 3\cos \theta)$$

$$= \cos^{-1}(\cos 3\theta)$$

$$= 3\theta$$

$$= 3\cos^{-1}x = \text{L.H.S}$$

Hence proved.

3.

**Write the function in the simplest form:**

$$\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}, x \neq 0.$$

$$\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}, x \neq 0.$$

$$\text{Ans - } \tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$$

By putting  $x = \tan \theta \rightarrow \theta = \tan^{-1} x$

$$\therefore \tan^{-1} \frac{\sqrt{1+x^2}-1}{x} = \tan^{-1} \frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta}$$

$$= \tan^{-1} \left( \frac{\sec \theta - 1}{\tan \theta} \right) = \tan^{-1} \left( \frac{1 - \cos \theta}{\sin \theta} \right)$$

$$\tan^{-1} \left( \frac{2 \sin^2 \left( \frac{\theta}{2} \right)}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right)$$

$$= \tan^{-1} \left( \tan \frac{\theta}{2} \right) = \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$$

4.

Write the function in the simplest form:

$$\tan^{-1} \left( \sqrt{\frac{1-\cos x}{1+\cos x}} \right), x < \pi.$$

**Ans** - Given that,  $\tan^{-1} \left( \sqrt{\frac{1-\cos x}{1+\cos x}} \right), x < \pi$

$$\begin{aligned} & \tan^{-1} \left( \sqrt{\frac{1-\cos x}{1+\cos x}} \right) \\ &= \tan^{-1} \left( \sqrt{\frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}}} \right) \\ &= \tan^{-1} \left( \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \right) \\ &= \tan^{-1} \left( \tan \frac{x}{2} \right) \\ &= \frac{x}{2} \end{aligned}$$

5.

**Write the function in the simplest form:**

$$\tan^{-1} \left( \frac{\cos x - \sin x}{\cos x + \sin x} \right), 0 < x < \pi.$$

**Ans** - We have given that,  $\tan^{-1} \left( \frac{\cos x - \sin x}{\cos x + \sin x} \right)$

$$\begin{aligned} &= \tan^{-1} \left( \frac{1 - \left( \frac{\sin x}{\cos x} \right)}{1 + \left( \frac{\sin x}{\cos x} \right)} \right) \\ &= \tan^{-1} \left( \frac{1 - \tan x}{1 + \tan x} \right) \\ &= \tan^{-1}(1) - \tan^{-1}(\tan x) \left[ \because \frac{-y}{x - xy} = \tan^{-1}x - \tan^{-1}y \right] \\ &= \frac{\pi}{4} - x \end{aligned}$$

6.

**Write the following function in the simplest form:**

$$\tan^{-1} \frac{x}{\sqrt{a^2-x^2}}, |x| < a.$$

**Ans** - We have given that,  $\tan^{-1} \frac{x}{\sqrt{a^2-x^2}}$ .

Let's consider,  $x = a \sin \theta \Rightarrow \frac{x}{a} = \sin \theta \Rightarrow \sin^{-1} \left( \frac{x}{a} \right)$

$$\therefore \tan^{-1} \frac{x}{\sqrt{a^2-x^2}}$$

$$= \tan^{-1} \frac{a \sin \theta}{\sqrt{a^2 - a^2 \sin^2 \theta}}$$

$$= \tan^{-1} \left( \frac{a \sin \theta}{a \sqrt{1 - \sin^2 \theta}} \right)$$

$$= \tan^{-1} \left( \frac{a \sin \theta}{a \cos \theta} \right)$$

$$= \tan^{-1}(\tan \theta) = \theta = \sin^{-1} \frac{x}{a}$$

7.

**Write the function in the simplest form:  $\tan^{-1} \left( \frac{3a^2x-x^3}{a^3-3ax^2} \right)$**

**Ans -** Consider,  $\tan^{-1} \left( \frac{3a^2x - x^3}{a^3 - 3ax^2} \right)$

Let's consider,  $x = a \tan \theta \Rightarrow \frac{x}{a} = \tan \theta \Rightarrow \theta = \tan^{-1} \left( \frac{x}{a} \right)$

$$\tan^{-1} \left( \frac{3a^2x - x^3}{a^3 - 3ax^2} \right)$$

$$= \tan^{-1} \left( \frac{3a^2 \cdot a \tan \theta - a^3 \tan^3 \theta}{a^3 - 3a \cdot a^2 \tan^2 \theta} \right)$$

$$= \tan^{-1} \left( \frac{3a^3 \tan \theta - a^3 \tan^3 \theta}{a^3 - 3a^3 \tan^2 \theta} \right)$$

$$= \tan^{-1}(\tan 3\theta)$$

$$= 3\theta$$

$$= 3 \tan^{-1} \frac{x}{a}$$

8.

**Find the value of  $\tan^{-1} \left[ 2 \cos \left( 2 \sin^{-1} \frac{1}{2} \right) \right]$**

**Ans** - Let's consider,  $\sin^{-1} \frac{1}{2} = x$

$$\text{Then } \sin x = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$$

$$\therefore \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

$$\therefore \tan^{-1} \left[ 2 \cos \left( 2 \sin^{-1} \frac{1}{2} \right) \right]$$

$$= \tan^{-1} \left[ 2 \cos \left( 2 \times \frac{\pi}{6} \right) \right]$$

$$= \tan^{-1} \left[ 2 \cos \frac{\pi}{3} \right]$$

$$= \tan^{-1} \left[ 2 \times \frac{1}{2} \right]$$

$$= \tan^{-1} 1 = \frac{\pi}{4}$$

9.

**Find the value of  $\tan \frac{1}{2} \left[ \sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2} \right]$ ,  $|x| <$**

**$1, y > 0$  and  $xy < 1$**

**Ans** - Let consider,  $x = \tan \theta$

Then,  $\theta = \tan^{-1}x$ .

$$\therefore \sin^{-1} \frac{2x}{1+x^2} = \sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right)$$

$$\sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2 \tan^{-1}x$$

Let's assume,  $y = \tan \theta$ . Then,  $\theta = \tan^{-1}y$ .

$$\therefore \cos^{-1} \left( \frac{1-y^2}{1+y^2} \right)$$

$$= \cos^{-1} \left( \frac{1-\tan^2 \theta}{1+\tan^2 \theta} \right)$$

$$= \cos^{-1}(\cos 2\theta)$$

$$= 2\theta = 2 \tan^{-1}y$$

$$\therefore \tan \frac{1}{2} \left[ \sin^{-1} \left( \frac{2x}{1+x^2} \right) + \cos^{-1} \left( \frac{1-y^2}{1+y^2} \right) \right]$$

$$= \tan \frac{1}{2} [2 \tan^{-1}x + 2 \tan^{-1}y]$$

$$\left[ \text{As, } \tan^{-1}x + \tan^{-1}y = \tan^{-1} \frac{x+y}{1+xy} \right]$$

$$= \tan \left[ \tan^{-1} \left( \frac{x+y}{1+xy} \right) \right]$$

$$= \frac{x+y}{1+xy}$$

10.

**Find the values of  $\sin^{-1} \left( \sin \frac{2\pi}{3} \right)$**

**Ans** - Let's consider,  $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$

As we know that  $\sin^{-1}(\sin x) = x$

If  $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ , which is the principle value branch of  $\sin^{-1}x$ .

Here,  $\frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Now,  $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$  can be written as:

$$\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$$

$$= \sin^{-1}\left[\sin\left(\pi - \frac{2\pi}{3}\right)\right]$$

$$= \sin^{-1}\left(\sin \frac{2\pi}{3}\right), \text{ where } \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore \sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \sin^{-1}\left(\sin \frac{\pi}{3}\right) = \frac{\pi}{3}$$

11.

**Find the values of  $\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$**

**Ans** - Let's consider,  $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

As we know that  $\tan^{-1}(\tan x) = x$

If  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , which is the principle value branch of  $\tan^{-1}x$ . Here,  $\frac{3\pi}{4} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ .

Now,  $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$  can be written as:

$$\begin{aligned}\tan^{-1}\left(\tan\frac{3\pi}{4}\right) &= \tan^{-1}\left[-\tan\left(\frac{-3\pi}{4}\right)\right] \\ &= \tan^{-1}\left[-\tan\left(\pi - \frac{\pi}{4}\right)\right]\end{aligned}$$

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[-\tan\left(\frac{-\pi}{4}\right)\right] \text{ where } -\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\therefore \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[-\tan\left(\frac{-\pi}{4}\right)\right] = \frac{-\pi}{4}$$

12.

**Find the value of  $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$**

**Ans** - Let consider,  $\sin^{-1} \frac{3}{5} = x$

$$\text{Then, } \sin x = \frac{3}{5}$$

$$\Rightarrow \cos x = \sqrt{1 - \sin^2 x} = \frac{4}{5}$$

$$\Rightarrow \sec x = \frac{5}{4}$$

$$\therefore \tan x = \sqrt{\sec^2 x - 1} = \sqrt{\frac{25}{16} - 1} = \frac{3}{4}$$

$$\therefore x = \tan^{-1} \frac{3}{4}$$

$$\therefore \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{3}{4} \dots \dots (i)$$

$$\text{Therefore, } \tan \left( \sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2} \right)$$

$$= \tan \left( \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{2}{3} \right) \text{ Using (i) and (ii)}$$

$$= \tan \left[ \tan^{-1} \left( \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \cdot \frac{2}{3}} \right) \right]$$

$$\left[ \text{As, } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x + y}{1 - xy} \right]$$

$$= \tan \left( \tan^{-1} \frac{9 + 8}{12 - 6} \right)$$

$$= \tan \left( \tan^{-1} \frac{17}{6} \right) = \frac{17}{6}$$

13.

**Find the values of  $\cos^{-1} \left( \cos \frac{7\pi}{6} \right)$  is equal to**

- (A)  $\frac{7\pi}{6}$  (B)  $\frac{5\pi}{6}$  (C)  $\frac{\pi}{3}$  (D)  $\frac{\pi}{6}$

**Ans** - As we know that  $\cos^{-1}(\cos x) = x$  if  $x \in [0, \pi]$ , which is the principle value branch of  $\cos^{-1}x$ . Here,  $\frac{7\pi}{6} \notin [0, \pi]$ .

Now,  $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$  can be written as:

$$\cos^{-1}\left(\cos \frac{7\pi}{6}\right) = \cos^{-1}\left(\cos \frac{-7\pi}{6}\right) = \cos^{-1}\left(2\pi - \frac{7\pi}{6}\right) \quad [\because +x = \cos x]$$

$$\therefore \cos^{-1}\left(\cos \frac{7\pi}{6}\right) = \cos^{-1}\left(\cos \frac{5\pi}{6}\right) = \frac{5\pi}{6}$$

The correct answer is B.

14.

**Find the values of  $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$  is equal to**

(A)  $\frac{1}{2}$  (B)  $\frac{1}{3}$  (C)  $\frac{1}{4}$  (D) 1

**Ans** - Let's consider,  $\sin^{-1}\left(-\frac{1}{2}\right) = x$ .

$$\text{Then, } \sin x = -\frac{1}{2} = -\sin \frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right)$$

As we know that the range of the Principal value branch of  $\sin^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .  $\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{6}$

$$\begin{aligned} \therefore \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) \\ = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1 \end{aligned}$$

The correct answer is D.

15.

**$\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$  is equal to**

(A)  $\pi$  (B)  $-\frac{\pi}{2}$  (C) 0 (D)  $2\sqrt{3}$

**Ans** - Suppose that,

$$A = \tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3}) \quad \dots (i)$$

As we know that,

$$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$$

So,

$$\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3})$$

Put the value in the equation (i)

$$A = \tan^{-1}(\sqrt{3}) - \pi + \cot^{-1}(-\sqrt{3})$$

As we know that,

$$\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$$

So,

$$\cot^{-1}(\sqrt{3}) = \frac{\pi}{2} - \tan^{-1}(\sqrt{3})$$

Put the value in the equation (ii),

$$A = \tan^{-1}(\sqrt{3}) - \pi + \frac{\pi}{2} - \tan^{-1}(\sqrt{3})$$

$$A = -\frac{\pi}{2}$$

Hence, the correct option is B.