

NCERT Solutions for Class 12 Maths

Chapter 13 – Probability

Chapter 13 – Probability Exercise 13.1

1.

Given that **E** and **F** are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$, find $P(E|F)$ and $P(F|E)$

Ans - Given that $P(E) = 0.6 = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{0.2}{0.3}$$

$$\Rightarrow P(E|F) = \frac{2}{3}$$

Similarly

$$P(F|E) = \frac{P(F \cap E)}{P(E)}$$

Now we know that $P(F \cap E)$ and $P(E \cap F)$ is same

$$\Rightarrow P(F|E) = \frac{0.2}{0.6}$$

$$\Rightarrow P(F|E) = \frac{1}{3}$$

$$\therefore P(E|F) = \frac{2}{3} \text{ and } P(F|E) = \frac{1}{3}$$

2.

Compute $P(A|B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$

Ans - Given $P(B) = 0.5$ and $P(A \cap B) = 0.32$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25}$$

Hence, we found that $P(A|B) = \frac{16}{25}$

3.

If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$ find

(i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(A \cup B)$

Ans - **(i)** Given that $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$$

$$\Rightarrow P(A \cap B) = 0.4 \times 0.8$$

$$\Rightarrow P(A \cap B) = 0.32$$

(ii) Given $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$

We know that $P(A|B)$ is the probability of occurrence of A when B has already happened

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now put $P(A \cap B) = 0.32$. $P(B) = 0.5$

$$\Rightarrow P(A|B) = \frac{0.32}{0.5}$$

$$\Rightarrow P(A|B) = 0.64$$

(iii) Given $P(A) = 0.8$, $P(B) = 0.5$, $P(B) = 0.5$ and $P(B|A) = 0.4$

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cup B) = 0.8 + 0.5 - 0.32$$

$$\Rightarrow P(A \cup B) = 0.98$$

4.

Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$

Ans - Given that $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$

$$\Rightarrow P(A) = \frac{5}{26}$$

We know, $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow \frac{2}{5} = \frac{13P(A \cap B)}{5} \quad \left(\text{since } P(B) = \frac{5}{13}, P(A|B) = \frac{2}{5} \right)$$

$$\Rightarrow P(A \cap B) = \frac{2}{13}$$

Also $P(A \cup B)$ is given by the formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$$

$$\Rightarrow P(A \cup B) = \frac{11}{26}$$

5.

If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$ find

(i) $P(A \cap B)$

(ii) $P(A|B)$

(iii) $P(B|A)$

Ans – (i) Given $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{7}{11} = \frac{6}{11} + \frac{5}{11} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{11}{11} - \frac{7}{11}$$

$$\Rightarrow P(A \cap B) = \frac{4}{11}$$

(ii) Given $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$

We know,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{4}{11} \times \frac{11}{5} \left(\text{since } P(A \cap B) = \frac{4}{11} \right)$$

$$\Rightarrow P(A|B) = \frac{4}{5}$$

(iii) Given $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$

We know,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{4}{11} \times \frac{11}{6}$$

$$\Rightarrow P(B|A) = \frac{2}{3}$$

Determine $P(E|F)$ in Exercises 6 to 9.

6.

A coin is tossed three times, where

(i) E: head on third toss, F: heads on first two tosses

(ii) E: at least two heads, F: at most two heads

(iii) E: at most two tails, F: at least one tail

Ans - Sample space is given by

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Events E and F are given by

$$E = \{HHH, HTH, THH, TTH\} \text{ and } F = \{HHH, HHT\}$$

$$\Rightarrow P(E) = \frac{4}{8} \text{ and } P(F) = \frac{2}{8}$$

$$\therefore E \cap F = \{HHH\}$$

$$\Rightarrow P(E \cap F) = \frac{1}{8}$$

Hence $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{1}{8}}{\frac{2}{8}}$$

$$\Rightarrow P(E|F) = \frac{1}{2}$$

(ii) Sample space is given by

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Events E and F are given by

$$E = \{HHH, HHT, THH, HTH\} \text{ and}$$

$$F = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$\Rightarrow P(E) = \frac{4}{8} \text{ and } P(F) = \frac{7}{8}$$

$$E \cap F = \{HHT, HTH, THH\}$$

$$\Rightarrow P(E \cap F) = \frac{3}{8}$$

Hence $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{3}{8}}{\frac{7}{8}}$$

$$\Rightarrow P(E|F) = \frac{3}{7}$$

(iii) Sample space is given by

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Events E and F are given by

$$E = \{HHH, HHT, THH, HTH, THH, THT, TTH\}$$

$$F = \{HHT, HTT, HTH, THH, THT, TTH, TTT\}$$

$$\Rightarrow P(E) = \frac{7}{8} \text{ and } P(F) = \frac{7}{8}$$

$$E \cap F = \{HHT, HTT, HTH, THH, THT, TTH\}$$

$$\Rightarrow P(E \cap F) = \frac{6}{8}$$

Hence $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{6}{8}}{\frac{7}{8}}$$

$$\Rightarrow P(E|F) = \frac{6}{7}$$

7.

Two coins are tossed once, where

(i) E: tail appears on one coin, F: one-coin shows head.

(ii) E: not tail appears, F: no head appears

Ans – (i) Sample Space is given by

$$S = \{HH, HT, TH, TT\}$$

Events E and F and their probabilities are given by

$$E = \{HT, TH\}$$

$$\Rightarrow P(E) = \frac{2}{4}$$

$$F = \{HT, TH\}$$

$$\Rightarrow P(F) = \frac{2}{4}$$

$$E \cap F = \{HT, TH\}$$

We know that $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{2}{4}}{\frac{2}{4}}$$

$$\Rightarrow P(E|F) = 1$$

That is, $(E|F)$ is a sure event

(ii) Sample Space is given by

$$S = \{HH, HT, TH, TT\}$$

Events E and F and their probabilities are given by

$$E = \{HH\}$$

$$\Rightarrow P(E) = \frac{1}{4}$$

$$F = \{TT\}$$

$$\Rightarrow P(F) = \frac{1}{4}$$

$$E \cap F = \phi$$

We know that $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{0}{\frac{1}{4}}$$

Thus, we found that $P(E|F) = 0$

8.

A die is thrown three times,

E : 4 appears on the third toss,

F : 6 and 5 appear respectively on the first two tosses

Ans - Number of elements in sample space is given by 216

Events E and F and their probabilities are given by

$$E = \left\{ \begin{array}{l} (1,1,4), (1,2,4) \dots \dots (1,6,4) \\ (2,1,4), (2,2,4) \dots \dots \dots (2,6,4) \\ (3,1,4), (3,2,4) \dots \dots \dots (3,6,4) \\ (4,1,4), (4,2,4) \dots \dots \dots (4,6,4) \\ (5,1,4), (5,2,4) \dots \dots \dots (5,6,4) \\ (6,1,4), (6,2,4) \dots \dots \dots (6,6,4) \end{array} \right\}$$

$$F = \{(6,5,1), (6,5,2), (6,5,3), (6,5,4), (6,5,5), (6,5,6)\}$$

$$\Rightarrow P(F) = \frac{6}{216}$$

$$\text{Hence } E \cap F = \{(6,5,4)\}$$

$$\Rightarrow P(E \cap F) = \frac{1}{216}$$

We know that $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{1}{216}}{\frac{6}{216}}$$

$$P(E|F) = \frac{1}{6}$$

9.

Mother, father and son line up at random for a family picture

E: son on one end,

F: father in middle

Ans - Let mother (M), father (F), and son (S) line up for the family picture, then the sample space will be as shown

$$A = \{MFS, MSF, FMS, FSM, SMF, SFM\}$$

Events E and F and their probabilities are

$$E = \{MFS, FMS, SMF, SFM\}$$

$$F = \{MFS, SFM\}$$

$$\Rightarrow P(F) = \frac{2}{6}$$

$$E \cap F = \{MFS, SFM\}$$

$$\Rightarrow P(E \cap F) = \frac{2}{6}$$

We know that $P(E|F)$ is given by

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{2}{6}}{\frac{2}{6}}$$

$$\Rightarrow P(E|F) = 1$$

10.

A black and a red dice are rolled

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Ans – (a) Let first observation come from the black die and the second from the red die respectively.

In case when two dices are rolled the elements in sample space is 36.

Events A and B and their probabilities are given by

$$A = \{(4,6), (5,5), (5,6), (6,4), (6,5), (4,6), (6,6)\}$$

Where A is the event when the sum is greater than 9.

Similarly,

$$B = \{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$$

Where B is the, event when black die resulted in a 5.

$$\Rightarrow P(B) = \frac{6}{36}$$

$$\text{Hence } A \cap B = \{(5,5), (5,6)\}$$

Conditional probability of obtaining a sum greater than 9, given that the black dice resulted in a 5 is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{2}{\frac{36}{6}}$$

$$\Rightarrow P(A|B) = \frac{1}{3}$$

(b) Let E and F be events and their probabilities defined as

E: Sum of the observations is 8

$$E = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

F: red die resulted in a number less than 4

$$F = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (2,1), (2,2), (2,2) \\ (3,1), (3,2), (3,3), (4,1), (4,2), (4,3) \\ (5,1), (5,2), (5,3), (6,1), (6,2), (6,3) \end{array} \right\}$$

$$\Rightarrow P(F) = \frac{18}{36}$$

$$E \cap F = \{(5,3), (6,2)\}$$

$$\Rightarrow P(E \cap F) = \frac{2}{36}$$

Conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4 is given by

than 4 is given $P(E|F)$ as shown

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{1}{9}$$

11.

A fair die is rolled. Consider events $E := \{1,3,5\}$, $F = \{2,3\}$ and $G = \{2, 3, 4, 5\}$. Find

(i) $P(E|F)$ and $P(F|E)$

(ii) $P(E|G)$ and $P(G|E)$

(iii) $P((E \cup F)|G)$ and $P((E \cap F)|G)$

Ans – (i) Sample space is given by

$$S = \{1,2,3,4,5,6\}$$

Given that $E = \{1,3,5\}$ and $F = \{2,3\}$

$$\Rightarrow E \cap F = \{3\}$$

$$\Rightarrow P(E \cap F) = \frac{1}{6}$$

$$\therefore P(E) = \frac{3}{6}$$

$$\therefore P(F) = \frac{2}{6}$$

Hence $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{1}{6}}{\frac{2}{6}} = \frac{1}{2}$$

Similarly, $P(F|E)$ is given by

$$P(F|E) = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

Thus $P(E|F) = \frac{1}{2}$ and $P(F|E) = \frac{1}{3}$

(ii) Given that $E = \{1,3,5\}$ and $G = \{2,3,4,5\}$

$$\therefore P(E) = \frac{3}{6}$$

$$\therefore P(G) = \frac{4}{6}$$

$$E \cap G = \{3,5\}$$

$$\Rightarrow P(E \cap G) = \frac{2}{6}$$

$P(E|G)$ is given by

$$P(E|G) = \frac{(E \cap G)}{P(G)}$$

$$\Rightarrow P(E|G) = \frac{\frac{2}{6}}{\frac{4}{6}}$$

$$\Rightarrow P(E|G) = \frac{1}{2}$$

Similarly $P(G|E)$ is given by

$$P(G|E) = \frac{\frac{2}{6}}{\frac{3}{6}}$$

$$\Rightarrow P(G|E) = \frac{2}{3}$$

Thus, $P(E|G) = \frac{1}{2}$ and $P(G|E) = \frac{2}{3}$

(iii) Sample space is given by

$$S = \{1,2,3,4,5,6\}$$

$$G = \{2,3,4,5\}$$

$$\text{We have } E \cup F = \{1,2,3,5\}$$

$$(E \cup F) \cap G = \{1,2,3,5\} \cap \{2,3,4,5\} = \{2,3,5\}$$

$$(E \cap F) \cap G = \{1,2,3,5\} \cap \{3\} = \{3\}$$

$$\therefore P(E \cup F) \cap G = \frac{3}{6}$$

$$\therefore P(E \cap F) \cap G = \frac{1}{6}$$

$$P(E \cup F | G) = \frac{P(E \cup F) \cap G}{P(G)}$$

$$\Rightarrow P(E \cup F | G) = \frac{\frac{3}{6}}{\frac{4}{6}}$$

$$\Rightarrow P(E \cup F | G) = \frac{3}{4}$$

Similarly

$$P(E \cap F | G) = \frac{\frac{1}{6}}{\frac{4}{6}}$$

$$\Rightarrow P(E \cap F | G) = \frac{1}{4}$$

Thus, $P(E \cup F | G) = \frac{3}{4}$ and $P(E \cap F | G) = \frac{1}{4}$

12.

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

(i) the youngest is a girl, (ii) at least one is a girl?

Ans – (i) sample space for a family having two children is given by

$$S = \{(B, B), (B, G), (G, G), (G, B)\}$$

Where B refers to boy child and G refers to girl child

Let an event be defined as

E: Both children are girls

$$E = \{(GG)\}$$

$$\therefore P(E) = \frac{2}{4}$$

Let F be an event defined as F: youngest child is girl

$$F = \{(BG), (GG)\}$$

$$\therefore E \cap F = \{(GG)\}$$

$$\therefore P(E \cap F) = \frac{1}{4}$$

We know that $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{1}{4}}{\frac{2}{4}}$$

$$\Rightarrow P(E|F) = \frac{1}{2}$$

(ii) Sample space for a family having two children is given by $S = \{(B, B), (B, G), (G, G), (G, B)\}$

Where B refers to boy child and G refers to girl child

Let an event be defined as.

E : Both children are girls

$$E = \{(GG)\}$$

$$\therefore P(E) = \frac{2}{4}$$

Let A be an event defined as

$$A = \{(B, G), (G, B), (G, G)\}$$

$$\therefore P(A) = \frac{3}{4}$$

$$\therefore E \cap A = \{(G, G)\}$$

$$\therefore P(E \cap A) = \frac{1}{4}$$

We know that

$P(E|A)$ is given by

$$P(E|A) = \frac{P(E \cap A)}{P(A)}$$

$$\Rightarrow P(E|A) = \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$\Rightarrow P(E|A) = \frac{1}{3}$$

13.

An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Ans - Given data can be represented as

	True/False	Multiple choice	Total
Easy	300	500	800
Difficult	200	400	600
Total	500	900	1400

Let us have following notations:-

E for easy questions

M for multiple questions

D for difficult questions

T for true/false questions

Given total number of questions is 1400, that of multiple questions is 900.

Hence probability for selecting easy multiple choice questions is given by

$$P(E \cap M) = \frac{500}{1400} = \frac{5}{14}$$

Probability for selecting multiple choice questions is given by

$$P(M) = \frac{9}{14} \text{ (since } \frac{90}{1400} = \frac{9}{14} \text{)}$$

$$\therefore P(E|M) = \frac{P(E \cap M)}{P(M)}$$

$$\Rightarrow P(E|M) = \frac{\frac{5}{14}}{\frac{9}{14}}$$

$$\Rightarrow P(E|M) = \frac{5}{9}$$

Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Ans - Let A and B be events defined as

A: the sum of the numbers on the dice is 4

B: the two numbers appearing on throwing the two dice are different.

$$\therefore A = \{(1,3), (2,3), (3,1)\}$$

$$\therefore B = \left\{ \begin{array}{l} (1,2), (1,3) \dots (1,6) \\ (2,1), (2,2) \dots (2,6) \\ (3,1), (3,2) \dots \dots (3,6) \\ (4,1), (4,2) \dots \dots (4,6) \\ (5,1), (5,2) \dots \dots \dots (5,6) \\ (6,1), (6,2) \dots \dots \dots (6,6) \end{array} \right\}$$

$$\Rightarrow P(B) = \frac{30}{36}$$

$$\therefore A \cap B = \{(1,3), (3,1)\}$$

$$P(A \cap B) = \frac{2}{36}$$

We know that $P(A|B)$ is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{\frac{2}{36}}{\frac{30}{36}}$$

$$\Rightarrow P(A|B) = \frac{1}{15}$$

15.

Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Ans - Sample space is given by

$$S = \{(1, H), (2, H), (1, T), (1, T), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, H), (4, T), (5, H), (5, T), (6, 1), (6, 2) \dots \dots (6, 6)\}$$

Let A and B be events defined as

A: The coin shows tail

B: at least one die shows 3

$$\therefore A = \{(1, T), (2, T), (4, T), (5, T)\}$$

$$\therefore B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)\}$$

$$\Rightarrow P(B) = \frac{7}{36}$$

Also, it is observable that $A \cap B = \phi$

We know that $P(A|B)$ given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{0}{\frac{7}{36}}$$

$$\Rightarrow P(A|B) = 0$$

16.

If $P(A) = \frac{1}{2}$ and $P(B) = 0$ then $P(A|B)$ is

- | | |
|----------------|------------------|
| A. 0 | B. $\frac{1}{2}$ |
| C. Not Defined | D. 1 |

Ans - Given that $P(A) = \frac{1}{2}$ and $P(B) = 0$

We know that $P(A|B)$ is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{0}$$

Thus $P(A|B)$ is not defined

Hence option C is the correct answer.

17.

If A and B are events such that $P(A|B) = P(B|A)$ then

(A) $A \subset B$ but $A \neq B$

(B) $A = B$

(C) $A \cap B = \phi$

(D) $P(A) = P(B)$

Ans - Given that

$$P(A|B) = P(B|A)$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow \frac{P(A)}{1} = \frac{P(B)}{1}$$

Thus, we found that $P(A) = P(B)$

Hence option D is the correct answer.

Exercise 13.2

1.

If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

Ans - Given that $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$

$P(A \cap B) = P(A) \cdot P(B)$, since A and B are independent

$$= \frac{3}{5} \cdot \frac{1}{5}$$

$$\Rightarrow P(A \cap B) = \frac{3}{25}$$

2.

Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Ans - It is known that in the pack of 52 cards, there are 26 black cards.

Let $P(A)$ denote the probability that a black card is drawn in the first draw.

$$\Rightarrow P(A) = \frac{26}{52} = \frac{1}{2}$$

Let suppose $P(B)$ denotes the probability that a black card is drawn in the second draw.

$$\Rightarrow P(B) = \frac{25}{51}$$

Hence, the probability that both the cards drawn are black

$$= \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

3.

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale

Ans - Let the events of choosing three oranges be A, B, and C respectively.

So, the probability that the orange drawn at first is good,

$$P(A) = \frac{12}{15}$$

Since, that orange is not replaced so, there are only 14 oranges left.

Similarly, the probability that the second orange chosen is good,

$$P(B) = \frac{11}{14}$$

Also, the probability that the third orange drawn is good

$$P(C) = \frac{10}{13}$$

Now, it is described that, if all three oranges are good, then the whole box will be ready to go for sale. Therefore, the probability that all the three oranges are good

$$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}$$

Hence, the probability that the whole box of oranges will be ready to go for sale is $\frac{44}{91}$.

4.

A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

Ans - Sample space when a fair coin and an unbiased coin are tossed is

$S =$

$\{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

Since for the event A, head comes on the fair coin, so

$A = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$

$$\Rightarrow P(A) = \frac{6}{12} = \frac{1}{2}$$

B suggests the event of appearing 3 in the die.

$B = \{(H, 3), (T, 3)\}$

$$\Rightarrow P(B) = \frac{2}{12} = \frac{1}{6}$$

Thus, $A \cap B = \{(H, 3)\}$ and so

$$P(A \cap B) = \frac{1}{12}$$

Now,

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} = P(A \cap B).$$

Hence, it is concluded that the events A and B are independent.

5.

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent?

Ans - Sample space when a die is thrown will be

$$S = \{1,2,3,4,5,6\}$$

So, when the number is even, then $A = \{2,4,6\}$

$$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$$

Also, when the number turns red, then $B = \{1,2,3\}$

$$\Rightarrow P(B) = \frac{3}{6} = \frac{1}{2}$$

Thus, $A \cap B = \{2\}$

$$\Rightarrow P(A \cap B) = \frac{1}{6}$$

$$\text{Now, } P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq \frac{1}{6}$$

That is, $P(A) \cdot P(B) \neq P(A \cap B)$

Hence, the events A and B are not independent.

6.

Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?

Ans - Given $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$.

$$\text{Now, } P(E) \cdot P(F) = \frac{3}{5} \times \frac{3}{10} = \frac{9}{50} \neq \frac{1}{5}$$

$$\therefore P(E) \cdot P(F) \neq P(E \cap F).$$

That is, $P(E) \cdot P(F) \neq P(E \cap F)$

Hence, the events E and F are not independent.

7.

Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are
(i) mutually exclusive (ii) independent

Ans – (i) It is known that if two events A and B are mutually exclusive, then

$$A \cap B = \phi$$

Therefore, $P(A \cap B) = 0$

Now, we also know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - 0$$

$$\Rightarrow p = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$$

(ii) Since, the events A and B are independent, so

$$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{a} p$$

Now, we know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - \frac{1}{2} p$$

$$\Rightarrow \frac{p}{2} = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$$

$$\Rightarrow p = \frac{2}{10} = \frac{1}{5}$$

8.

Let A and B be independent events with $P(A) = 0.3$, $P(B) = 0.4$. Find

(i) $P(A \cap B)$ (ii) $P(A \cup B)$

(iii) $P(A|B)$ (iv) $P(B|A)$

Ans – (i) Given $P(A) = 0.3$ and $P(B) = 0.4$.

Since the two events are independent, so

$$P(A \cap B) = P(A) \cdot P(B) = 0.3 \times 0.4 = 0.12$$

(ii) We know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Since $P(A \cap B) = 0.12$,

$$P(A \cup B) = 0.3 + 0.4 - 0.12 = 0.58$$

$$\text{(iii) } P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{0.12}{0.3} = 0.4$$

$$\text{(iv) } P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{0.12}{0.3} = 0.4$$

9.

If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find $P(\text{not A and not B})$

Ans - Given $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$.

We know, $P(\{A\}' \cap \{B\}')$ = $P(A \cup B)'$

$$\Rightarrow P(\{A\}' \cap \{B\}')$$
 = $1 - P(A \cup B)$.

Again, it is known that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(\{A\}' \cap \{B\}')$$
 = $1 - P(A \cup B)$
= $1 - [P(A) + P(B) - P(A \cap B)]$

$$= 1 - \left[\frac{1}{4} + \frac{1}{2} - \frac{1}{8} \right]$$

$$= 1 - \frac{5}{8}$$

$$= \frac{3}{8}$$

Hence, $P(\text{not A and not B}) = \frac{3}{8}$

10.

Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$, and $P(A' \cap B') = \frac{1}{4}$. State whether A and B are independent?

Ans - Given $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\{A\}' \cup \{B\}') = \frac{1}{4}$

It is known that, $P(\{A\}' \cap \{B\}') = P(A \cup B)'$

$$\Rightarrow P(\{A\}' \cap \{B\}') = \frac{1}{4}$$

$$\Rightarrow P(A \cap B)' = \frac{1}{4}$$

$$\Rightarrow P(A \cap B) = \frac{3}{4}$$

$$\text{But, } P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$$

Therefore, $P(A) \cdot P(\bar{B}) \neq P(A \cap B)$.

Hence, the events A and B are not independent.

11.

Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find

(i) $P(A \text{ and } B)$

(ii) $P(A \text{ and not } B)$

(iii) $P(A \text{ or } B)$

(iv) $P(\text{neither } A \text{ nor } B)$

Ans - **(i)** Since, the events A and B are independent, so

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$= 0.3 \times 0.6 = 0.18$$

$$\text{(ii) } P(A \text{ and not } B) = P(A \cap \{B\}') = P(A) - P(A \cap B)$$

$$= 0.3 - 0.18$$

$$= 0.12$$

(iii) It is known that, $P(A \text{ or } B) = P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= 0.3 + 0.6 - 0.18$$

$$= 0.72$$

(iv) It is known that, $P(\text{neither } A \text{ nor } B) = P(A \cup B)'$

$$= 1 - P(A \cup B)$$

$$= 1 - 0.72 = 0.28$$

12.

A die is tossed thrice. Find the probability of getting an odd number at least once.

Ans - Probability of having an odd number in tossing the die one times $= \frac{3}{6} = \frac{1}{2}$

Again, probability of having an even number $= \frac{3}{6} = \frac{1}{2}$

Therefore, probability of having an even number thrice

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Thus, probability of obtaining an odd number at least one times.

$= 1 -$ probability of not having an odd number in any of the throws

$= 1 -$ probability of having an even number three times

$$= 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$

13.

Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

(i) both balls are red.

(ii) first ball is black and second is red.

(iii) one of them is black and other is red.

Ans – (i) There are total 18 balls, among them 8 are red and 10 are black.

Therefore, the probability of having a red ball on drawing the first time = $\frac{8}{18} = \frac{4}{9}$.

The total count of balls remains the same as the balls are replaced.

So, the probability of having red ball on drawing in the second draw = $\frac{8}{18} = \frac{4}{9}$.

Thus, the probability of having both the ball is red

$$= \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$$

(ii) Probability of having a black ball in first draw = $\frac{10}{18} = \frac{5}{9}$.

Total count of balls remains the same as the balls are replaced.

Then, the probability of having a red ball in the second draw = $\frac{8}{18} = \frac{4}{9}$

Thus, the probability of having the black and red balls in the first and second draw respectively

$$= \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$$

(iii) Probability of having a red ball in first draw = $\frac{8}{18} = \frac{4}{9}$

Total count of balls remains the same as the balls are replaced.

Then, Probability of having a black ball in the second draw
 $= \frac{10}{18} = \frac{5}{9}$

Thus, the probability of having the red and blackballs in the first and second draw respectively.

$$= \frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$$

⇒ Probability of having one ball as red and the other as black = probability of obtaining the first ball is black and the other is red + probability of obtaining the first ball is red and the other is black

$$= \frac{20}{81} + \frac{20}{81}$$
$$= \frac{40}{81}$$

14.

Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem.

Ans - (i) Probability that A can solve the problem, $P(A) = \frac{1}{2}$

Probability that B can solve the problem, $P(B) = \frac{1}{3}$

Given that the problem is solved by them independently.

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Also,

$$P(\{A\}^c) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2} \text{ and}$$

$$P(\{B\}^c) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$$

Thus, the probability that A or B can solve the problem

$$\begin{aligned} &= P(A \cup B) \\ &= P(A) + P(B) - P(A \cap B) \\ &= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} \\ &= \frac{4}{6} = \frac{2}{3} \end{aligned}$$

That is, the probability that the problem is solved = $\frac{2}{3}$.

(ii) Probability that exactly one of A and B solves the problem will be,

$$\begin{aligned} &P(A) \cdot P(\{B\}^c) + P(B) \cdot P(\{A\}^c) \\ &= \frac{1}{2} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{2} \\ &= \frac{1}{3} + \frac{1}{6} \\ &= \frac{1}{2} \end{aligned}$$

15.

One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent ?

(i) E : 'the card drawn is a spade'

F : 'the card drawn is an ace'

(ii) E : 'the card drawn is black'

F : 'the card drawn is a king'

(iii) E : 'the card drawn is a king or queen'

F : 'the card drawn is a queen or jack'.

Ans - (i) It is known that there are 13 spade and 4 ace cards in a deck of 52 cards.

Therefore, the probability that the card chosen is a spade

$$P(E) = \frac{13}{52} = \frac{1}{4}$$

Also, the probability that the card chosen is an ace

$$P(F) = \frac{4}{52} = \frac{1}{13}$$

Since only one card is an ace as well as a spade.sc

$$P(EF) = \frac{1}{52}$$

$$\text{Now, } P(E) \cdot P(F) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} = P(EF).$$

Hence E and F are independent events.

(ii) It is known that there are 26 black and 4 king cards in a deck of 52 cards.

Therefore, the probability that the card chosen is black,

$$P(E) = \frac{26}{52} = \frac{1}{2}$$

Also, the probability that the card chosen is a king

$$P(F) = \frac{4}{52} = \frac{1}{13}$$

Since only two cards are black as well as king, so

$$P(EF) = \frac{2}{52} = \frac{1}{26}$$

$$\text{Now, } P(E) \cdot P(F) = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26} = P(EF)$$

Hence, E and F are independent events.

(iii) It is known that there are 4 king, 4 queen, and 4 jack cards in a deck of 52 cards. Therefore, the probability that the card chosen is a king or queen

$$P(E) = \frac{8}{52} = \frac{2}{13}$$

Also, the probability that the card chosen is a queen or jack,

$$P(F) = \frac{8}{52} = \frac{2}{13}$$

Since only 4 cards are a king or a queen, or queen or a jack, so

$$P(EF) = \frac{4}{52} = \frac{1}{13}$$

$$\text{Now, } P(E) \cdot P(F) = \frac{2}{13} \times \frac{2}{13} = \frac{4}{169} \neq \frac{1}{13}$$

Therefore, $P(E) \cdot P(F) \neq P(EF)$

Hence, E and F are not independent events.

In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Ans – (a) Let H and E denote the students who read Hindi newspapers and the student who read English newspapers respectively, it is provided that,

$$P(H) = 60 = \frac{3}{5}$$

$$P(E) = 40, \text{ and}$$

$$P(H \cap E) = 20$$

Probability that a student reads neither Hindi nor English newspaper is,

$$P(H \cup E)' = 1 - P(H \cup E) = 1 - \{P(H) + P(E) - P(H \cap E)\}$$

$$= 1 - \left(\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right)$$

$$= 1 - \frac{4}{5}$$

$$= \frac{1}{5}$$

(b) Probability that a student who reads an English newspaper also reads Hindi newspapers,

$$P(E|H) = \frac{P(E \cap H)}{P(H)}$$

$$= \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$$

(c) Probability that a student reads Hindi newspaper also reads English newspapers,

$$P(E|H) = \frac{P(E \cap H)}{P(H)}$$

$$= \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}$$

17.

The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

- (A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{12}$ (D) $\frac{1}{36}$

Ans - The sample space contains a total of 36 outcomes when two dice are thrown.

2 is the only even number which is prime.

Therefore, if E be the event of obtaining an even prime number on each die, then $E = \{(2,2)\}$

$$\Rightarrow P(E) = \frac{1}{36}$$

Hence correct answer is option D.

18.

Two events A and B will be independent, if

(A) A and B are mutually exclusive

(B) $P(A'B') = [1 - P(A)][1 - P(B)]$

(C) $P(A) = P(B)$

(D) $P(A) + P(B) = 1$

Ans - Correct Answer:- Option (B)

It is known that two events A and B are independent, if

$$P(AB) = P(A) \times P(B)$$

$$\text{Now, } P(\{A\}'\{B\}') = [1 - P(A)][1 - P(B)]$$

$$\Rightarrow P(\{A\}' \cap \{B\}') = 1 - P(A) - P(B) + P(A) \cdot P(B)$$

$$\Rightarrow 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A)P(B)$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)P(B)$$

$$\Rightarrow P(A) + P(B) - P(AB) = P(A) + P(B) - P(A)P(B)$$

$$\Rightarrow P(AB) = P(A) \cdot P(B)$$

Hence, this concludes that A and B are independent events.

Hence correct answer is option B.

Incorrect Answers

Option (A)

Now, let $P(A) = m$ and $P(B) = n$ where $0 < m, n < 1$.

If possible, let A and B are two mutually exclusive events

$$\text{Therefore, } A \cap B = \phi$$

$$\Rightarrow P(AB) = 0 \text{ But, } P(A) \cdot P(B) = mn \neq 0$$

$$\text{Thus, } P(A) \cdot P(B) \neq P(AB)$$

that is, A and B are not independent events

Hence, option A is incorrect

Option (C)

Let consider two events A and B such that

$A = \{1,3,5\}$ $B = \{2,4,6\}$ and the sample space
 $S = \{1,2,3,4,5,6\}$

Therefore, $P(A) = \frac{3}{6} = \frac{1}{2}$ and $P(B) = \frac{3}{6} = \frac{1}{2}$

that is, $P(A) = P(B)$

Also, $A \cap B = \{\phi\}$ and so, $P(A \cap B) = 0$

That is, $P(AB) = 0$

Now, $P(A)P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq 0$

Thus, $P(A) \cdot P(B) \neq P(AB)$

\Rightarrow A and B are not independent events.

Hence, option C is incorrect

Option (D)

Considering the previous example, it can be seen that

$P(A) + P(B) = \frac{1}{2} + \frac{1}{2} = 1$ but has been proved that the events A and B are no independent.

Hence, the option D is incorrect.

Exercise 13.3

1.

An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Ans - Given the urn contains 5 red and 5 black balls.

Let a red ball be drawn in the first attempt.

Then, probability of drawing a red ball $= \frac{5}{10} = \frac{1}{2}$

Now, we add two more red balls. Therefore, the urns contain 7 red and 5 black ball.

Probability of drawing the second red ball $= \frac{7}{12}$

Let a black ball be drawn in the first attempt.

Then, probability of drawing a black ball $= \frac{5}{10} = \frac{1}{2}$

Now, we add two more red balls. Therefore, the urns contain 5 red and 7 black balls. Probability of drawing the second red ball $= \frac{5}{12}$

Therefore, total probability of drawing second ball as red is

$$\begin{aligned} &= \frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12} = \frac{1}{2} \left(\frac{7}{12} + \frac{5}{12} \right) \\ &= \frac{1}{2} \times 1 = \frac{1}{2} \end{aligned}$$

2.

A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

Ans - Let A be the event of getting a red ball.

Let E_1 and E_2 be the events of selecting the first bag and second bag respectively.

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$\Rightarrow P(A|E_1) = P(\text{drawing a red ball from first bag}) = \frac{4}{8} = \frac{1}{2}$$

$$\Rightarrow P(A|E_2) = P(\text{drawing a red ball from first bag}) = \frac{2}{8} = \frac{1}{4}$$

Probability of drawing a ball from the first bag, given that it is red, is given by $P(E_1|A)$

By using Bayes theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}}$$

$$= \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{8}} = \frac{2}{3}$$

3.

Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

Ans - Let E_1 and E_2 be the events where the student is a hostler and a day scholar respectively.

Let A be the event that the chosen students get a grade A.

$$P(E_1) = 60$$

$$P(E_2) = 40$$

$$\Rightarrow P(A|E_1) = P(\text{student getting an A grade is a hostler}) = 30$$

$$\Rightarrow P(A|E_2) = P(\text{student getting an A grade is a day scholar}) = 20$$

The probability that a randomly chosen student is a hostler, given that he has an A grade, is given by $P(E_1|A)$

By using Bayes theorem, we get,

$$\begin{aligned} P(E_1|A) &= \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} \\ &= \frac{0.6 \times 0.3}{0.6 \times 0.3 + 0.4 \times 0.2} \\ &= \frac{0.18}{0.26} \\ &= \frac{9}{13} \end{aligned}$$

4.

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Ans - Let E_1 and E_2 be the events in which the student knows the answer and the student guesses the answer respectively.

$$P(E_1) = \frac{3}{4}$$

$$P(E_2) = \frac{1}{4}$$

$$\Rightarrow P(A|E_1)$$

= P(student answer correctly, given he knows the answer)

$$= 1$$

$$\Rightarrow P(A|E_2)$$

= P(student answer correctly, given that he guessed) = $\frac{1}{4}$

Probability that the student knows the answer, given that the answer is correct, is given by $P(E_1|A)$.

By using Bayes theorem, we get,

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{3}{4} \times 1}{\frac{3}{4} \times 1 + \frac{1}{4} \times \frac{1}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}}$$

$$= \frac{3}{4} \div \frac{13}{16}$$

$$= \frac{12}{13}$$

5.

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

Ans - Let E_1 and E_2 be the respective events that a person has a disease and a person has no disease

$$P(E_1) = 0.1$$

Since E_1 and E_2 are events complementary to each other

$$P(E_2) = 1 - P(E_1) = 1 - 0.001 = 0.999$$

Let A be the event that the answer is correct.

$$\Rightarrow P(A|E_1) = P(\text{result is positive given the person has disease}) = 99$$

$$\Rightarrow P(A|E_2) = P(\text{result is positive given that the person has no disease}) = 0.5$$

The probability that a person has a disease, given that his test result is positive, is given by $P(E_1|A)$

By using Bayes theorem, we get,

$$\begin{aligned} P(E_1|A) &= \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} \\ &= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005} \\ &= \frac{0.00099}{0.00099 + 0.004995} \\ &= \frac{0.00099}{0.005985} \\ &= \frac{22}{133} \end{aligned}$$

6.

There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

Ans - Let E_1 , E_2 and E_3 be the events of choosing a two headed coin, a biased coin, and an unbiased coin respectively.

Let A be the event that the coin shows heads

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

A two-headed coin will always show heads

$$\Rightarrow P(A|E_1) = P(\text{coin showing heads, given that it is two-headed coin}) = 1$$

Probability of getting heads, given that coin is biased = 75

$$\Rightarrow P(A|E_2) = P(\text{getting heads, given that coin is biased})$$

$$= \frac{75}{100} = \frac{3}{4}$$

The third coin is unbiased hence the probability of getting heads is

$$= \frac{1}{2}$$

$$\Rightarrow P(A|E_3)$$

$$= P(\text{getting heads, given that the coin is unbiased})$$

$$= \frac{1}{2}$$

$$\Rightarrow P(A|E_2) = P(\text{car driver met with an accident})$$

$$= 0.03 = \frac{3}{100}$$

$$\Rightarrow P(A|E_3) = P(\text{truck driver met with accident})$$

$$= 0.15 = \frac{1}{100}$$

The probability that the driver is a scooter driver, given that he met with an accident, is given by $P(E_1|A)$

By using, Bayes theorem, we get

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{15}{100}}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{100} \left(\frac{1}{6} + 1 + \frac{15}{2} \right)}$$

$$= \frac{1}{6} \div \frac{104}{12}$$

$$= \frac{1}{52}$$

7.

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Ans - Let E_1 , E_2 and E_3 be the events that the driver is a scooter driver, a car driver, and a truck driver respectively.

Total number of drivers = 2000 Scooter drivers + 44000 car drivers + 6000 truck drivers = 12000 drivers

$$\therefore P(E_1) = \frac{2000}{12000} = \frac{1}{6}$$

$$P(E_2) = \frac{4000}{12000} = \frac{1}{3}$$

$$PP(E_3) = \frac{6000}{12000} = \frac{1}{2}$$

Let A be the event that the person meets with an accident

$$\Rightarrow P(A|E_1) = P(\text{scooter driver met with an accident})$$

$$= 0.01 = \frac{1}{100}$$

$$\Rightarrow P(A|E_2) = P(\text{car driver met with an accident})$$

$$= 0.03 = \frac{3}{100}$$

$$\Rightarrow P(A|E_3) = P(\text{truck driver met with accident})$$

$$= 0.15 = \frac{1}{100}$$

The probability that the driver is a scooter driver, given that he met with an accident, is given by $P(E_1|A)$

By using, Bayes theorem, we get

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{15}{100}}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{100} \left(\frac{1}{6} + 1 + \frac{15}{2} \right)}$$

$$= \frac{1}{6} \div \frac{104}{12}$$

$$= \frac{1}{52}$$

8.

A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

Ans - Let E_1 and E_2 be the respective events of items produced by machines A and B

$$: P(E_1) = 60$$

$$P(E_2) = 40$$

Let A be the event that the produced items were found to be defective.

$$\Rightarrow P(A|E_1) = P(\text{product is defective, given that machine A produced}) = 2$$

$$\Rightarrow P(A|E_2) = P(\text{product is defective, given that machine B produced}) = 1$$

The probability that the randomly selected items was from B, given that it is defective, is given by $P(E_2|A)$

By using Bayes theorem, we get,

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{2}{5} \times \frac{1}{100}}{\frac{3}{5} \times \frac{2}{100} + \frac{2}{5} \times \frac{1}{100}}$$

$$= \frac{\frac{2}{500}}{\frac{6}{500} + \frac{2}{500}} = \frac{2}{8}$$

$$= \frac{1}{4}$$

9.

Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Ans - Let E_1 and E_2 be respective events in which the first group and the second group win the competition.

$$P(E_1) = 0.6$$

$$P(E_2) = 0.4$$

Let A be event of introducing a new product.

$$\Rightarrow P(A|E_1) = P(\text{introducing a new product, given that first group wins}) = 0.7$$

$$\Rightarrow P(A|E_2) = P(\text{introducing a new product, given that second group wins}) = 0.3$$

Probability that new product is introduced by the second group, is given by $P(E_2|A)$

By using Bayes theorem, we get,

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{0.4 \times 0.3}{0.6 \times 0.7 + 0.4 \times 0.3}$$

$$= \frac{0.12}{0.42 + 0.12}$$

$$= \frac{0.12}{0.54}$$

$$= \frac{12}{54}$$

$$= \frac{2}{9}$$

10.

Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

Ans - Let E_1 be event that the outcome on the die is 5 or 6 and E_2 be event that the outcome on the die is 1, 2, 3, or 4.

$$P(E_1) = \frac{2}{6} = \frac{1}{3}$$

$$P(E_2) = \frac{4}{6} = \frac{2}{3}$$

Let A be event of getting exactly one head.

$P(E_1)$ is same as tossing the coin 3 times.

$P(E_2)$ is same as tossing the coin exactly once

$\Rightarrow P(A|E_1) = P(\text{getting exactly one head, given the coin is tossed 3 times}) = \frac{3}{8}$ (TTH, THT, HTT)

$\Rightarrow P(A|E_2) = P(\text{getting exactly one head, given the coin is tossed only once}) = \frac{1}{2}$

Probability that the girl threw 1,2,3, or 4 with die, if she obtained exactly one head, is given by $P(E_2|A)$

By using Bayes theorem, we get

$$\begin{aligned}P(E_2|A) &= \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} \\&= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} \\&= \frac{\frac{1}{3}}{\frac{1}{3} \left(\frac{3}{8} + 1 \right)} \\&= \frac{1}{11} \div 8 \\&= \frac{8}{11}\end{aligned}$$

11.

A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Ans - Let E_1 , E_2 and E_3 be events that the time consumed by machine operator A, B and C on the job

$$P(E_1) = 50$$

$$P(E_2) = 30$$

$$P(E_3) = 20$$

Let A be event of producing defective items

$$\Rightarrow P(A|E_1) = P(\text{item is defective, given that it is produced by A}) = 1$$

$$\Rightarrow P(A|E_2) = P(\text{item is defective, given that it is produced by B}) = 5$$

$$\Rightarrow P(A|E_3) = P(\text{item is defective, given that it is produced by C}) = 7$$

Probability that the defective item was produced by A is given by, is given by $P(E_1|A)$

By using Bayes theorem, we get,

$$\begin{aligned} P(E_1|A) &= \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)} \\ &= \frac{\frac{1}{2} \times \frac{1}{100}}{\frac{1}{2} \times \frac{1}{100} + \frac{3}{10} \times \frac{5}{100} + \frac{1}{5} \times \frac{7}{100}} \\ &= \frac{\frac{1}{100} \times \frac{1}{2}}{\frac{1}{100} \left(\frac{1}{2} + \frac{3}{2} + \frac{7}{2} \right)} \\ &= \frac{1}{2} \div \frac{17}{5} = \frac{5}{34} \end{aligned}$$

12.

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Ans - Let E_1 and E_2 be respective events of choosing a diamond card and a card which is not diamond.

Let A be the event that denote the lost card.

$$P(E_1) = \frac{13}{52} = \frac{1}{4}$$

$$P(E_2) = \frac{39}{52} = \frac{3}{4}$$

When the card lost is a diamond card, then there are 12 diamond cards out of 51 cards. From 12 diamond cards 2 cards can be drawn in ${}^{12}C_2$ ways.

\therefore Probability of getting two diamond cards, when one diamond card is lost, is given by $P(A|E_1)$

$$P(A|E_1) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12!}{2! \times 10!} \times \frac{2! \times 49!}{5!} = \frac{22}{425}$$

When the card lost is not a diamond card, then there are 13 diamond cards out of 51 cards.

From 13 diamond cards 2 cards can be drawn in ${}^{13}C_2$ ways

\therefore Probability of getting two diamond cards, when the card lost is not a diamond, is given by $P(A|E_2)$

$$P(A|E_2) = \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{13!}{2! \times 11!} \times \frac{2! \times 49!}{5!} = \frac{26}{425}$$

Probability that the lost card is diamond is given by $P(E_1|A)$

$$\begin{aligned} P(E_1|A) &= \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} \\ &= \frac{\frac{1}{4} \times \frac{22}{425}}{\frac{1}{4} \times \frac{22}{425} + \frac{3}{4} \times \frac{26}{425}} \\ &= \frac{11}{2} \div 25 = \frac{11}{50} \end{aligned}$$

13.

Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is

- A) $\frac{4}{5}$ B) $\frac{1}{2}$ C) $\frac{1}{5}$ D) $\frac{2}{5}$

Ans - Let E_1 and E_2 be events such that

E_1 : A speaks truth

E_2 : A speaks false

Let X be the event that a head appears

$$P(E_1) = \frac{4}{5}$$

$$\therefore P(E_2) = 1 - P(E_1) = 1 - \frac{4}{5} = \frac{1}{5}$$

If a coin is tossed, then it may result in either head(H) or tail(T).

Probability of getting a head is $\frac{1}{2}$ whether speak truth or not.

$$\therefore P(X|E_1) = P(X|E_2) = \frac{1}{2}$$

$$P(E_1|X) = \frac{P(E_1)P(X|E_1)}{P(E_1)P(X|E_1) + P(E_2)P(X|E_2)}$$

$$= \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{4}{5} \times \frac{1}{2} + \frac{1}{5} \times \frac{1}{2}}$$

$$= \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{1}{2} \left(\frac{4}{5} + \frac{1}{5} \right)}$$

$$= \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{1}{2} \times 1} = \frac{4}{5}$$

Hence correct answer is option A.

If A and B are two events such that $A \subset B$ and $P(B) \neq 0$ then which of the following is correct?

A) $P(A|B) = \frac{P(B)}{P(A)}$

B) $P(A|B) < P(A)$

C) $P(A|B) \geq P(A)$

D) None of these

Ans - If $A \subset B$ then $A \cap B = A$

$$\Rightarrow A \cap B = P(A)$$

Also, $P(A) < P(B)$

$$\text{Let } P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \neq \frac{P(B)}{P(A)} \quad \dots (1)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \quad \dots (2)$$

It is known that, $P(B) \leq 1$.

From (2), we obtain

$$\Rightarrow \frac{1}{P(B)} \geq 1$$

$$\Rightarrow \frac{P(A)}{P(B)} \geq P(A)$$

From (2), we obtain

$$\Rightarrow P(A|B) \geq P(A) \quad \dots (3)$$

$\therefore P(A|B)$ is not less than $P(A)$.

Thus, from (3), it can be concluded that the relation given in alternate C is correct.

Hence correct answer is option C.

Miscellaneous Exercise

1.

If A and B are two events such that $P(A) \neq 0$ Find $P(B|A)$

(i) A is a subset of B (ii) $A \cap B = \phi$

Ans – (i) Given that $P(A) \neq 0$ and A is a subset of B

$$\Rightarrow P(A \cap B) = P(A)$$

$$\Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\therefore P(B|A) = \frac{P(A)}{P(A)} = 1$$

(ii) Since it is given that $A \cap B = \phi$

$$\Rightarrow P(A \cap B) = 0$$

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = 0$$

2.

A couple has two children,

(i) Find the probability that both children are males, if it is known that at least one of the children is male.

(ii) Find the probability that both children are females, if it is known that the elder child is a female.

Ans - (i) Sample space for a family to have two children is given by $S = \{BB, GG, BG, GB\}$

Let us have the following notations and their probabilities given as shown

N: Both children are males

$$P(N) = \frac{1}{4}$$

K: at least one of the children is male

$$P(K) = \frac{3}{4}$$

$$\therefore N \cap K = BB$$

$$\therefore P(N \cap K) = \frac{1}{4}$$

Now probability that both of the children are males provided at least one of the child is male is given by

$$P(N|K) = \frac{P(N \cap K)}{P(K)}$$

$$P(N|K) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Hence the probability that both of the children are males provided at least one of the children is male is $\frac{1}{3}$

(ii) Sample space for a family to have two children is given by $S = \{BB, GG, BG, GB\}$

Let us have the following notations and their probabilities given as shown

N : Both children are females

$$P(N) = \frac{1}{4}$$

K : the elder child is a female

$$P(K) = \frac{2}{4}$$

$$\therefore N \cap K = GG$$

$$\therefore P(N \cap K) = \frac{1}{4}$$

Now probability that both of the children are males provided at least one of the children is male is given by

$$P(N|K) = \frac{P(N \cap K)}{P(K)}$$

$$P(N|K) = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$$

Hence the probability that both of the children are males provided at least one of the children is male is $\frac{1}{2}$.

3.

Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

Ans - Given that 5% of men and 0.25% of women have grey hair.

$$\Rightarrow \text{Total people having grey hair} = (5 + 0.25) = 5.25\%$$

$$\therefore \text{Probability of male being haired} = \frac{5}{5.25}$$

4.

Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Ans - Given probability of a person being right-handed,
 $p = 0.9$

Since a person can only be right-handed or left handed.
Therefore it follows a binomial distribution with

$$n = 10, p = \frac{9}{10}, q = \frac{1}{10}$$

Probability that at least 6 people are right handed is given by

$$\sum_{k=0}^{k=7} {}^{10}C_k \left(\frac{9}{10}\right)^k \left(\frac{1}{10}\right)^{10-k}$$

Therefore, probability that at most 6 people are right handed is given by

$$1 - \sum_{k=0}^{k=7} {}^{10}C_k \left(\frac{9}{10}\right)^k \left(\frac{1}{10}\right)^{10-k}$$

5.

If a leap year is selected at random, what is the chance that it will contain 53 tuesdays?

Ans - In a leap year, we have 366 days i.e., 52 weeks and 2 days.

⇒ In 52 weeks, we have 52 Tuesdays.

⇒ Probability that leap year will contain 53 Tuesdays = probability that remaining 2 days will be Tuesdays.

Remaining 2 days can be any of the following :- {Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday, Sunday and Monday}

∴ Total number of cases = 7

∴ Probability that a leap year will have 53 Tuesdays = $\frac{2}{7}$

6.

Suppose we have four boxes. A, B, C and D contain coloured marbles as given below.

Box	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

Ans: Let us have the following notations

R be the event of drawing a red marble

E be the event of selecting the box A

F be the event of selecting the box B

G be the event of selecting the box C

Total number of marbles = 40

Number of red marbles = 15

$$P(R) = \frac{15}{40}$$

Number of red marbles in box A i.e $n(R \cap E) = 1$

Number of red marbles in box B i.e $n(R \cap F) = 6$

Number of red marbles in box C i.e $n(R \cap G) = 8$

Now probability that red marble is picked from box A is given by

$$P(R|E) = \frac{P(R \cap E)}{P(E)}$$

$$\Rightarrow P(R|E) = \frac{\frac{1}{40}}{\frac{15}{40}} = \frac{1}{15}$$

Hence E be the event of selecting the box A is $\frac{1}{15}$

Similarly, probability that red marble is picked from box B is given by

$$P(R|F) = \frac{\frac{6}{40}}{\frac{15}{40}} = \frac{2}{5}$$

Hence probability that red marble is picked from box B is $\frac{2}{5}$.

Similarly, probability that red marble is picked from box C is given by

$$P(R|G) = \frac{\frac{8}{40}}{\frac{15}{40}} = \frac{8}{15}$$

Hence probability that red marble is picked from box C is $\frac{8}{15}$

7.

Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Ans - Let us have the following notations

E: Events when person took yoga and meditation courses

$$\therefore P(E) = \frac{1}{2}$$

F: The events when the person took drugs

$$\therefore P(F) = \frac{1}{2}$$

G: the person suffered a heart attack

$$\therefore P(G) = 0.4$$

From the question also, we have

$$P(G|E) = 0.4 \times 0.7 = 0.28$$

$$P(G|F) = 0.4 \times 0.7 = 0.30$$

Now probability that found person has a heart attack despite having yoga and meditation courses is given by

$$P(E|G) = \frac{P(E) \times P(G|E)}{P(E) \times P(G|E) + P(F) \times P(G|F)}$$

$$P(E|G) = \frac{0.5 \times 0.28}{0.5 \times 0.28 + 0.5 \times 0.3} = \frac{14}{29}$$

Hence the probability that the found person has a heart attack despite having yoga and meditation courses is $\frac{14}{29}$.

8.

If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$).

Ans - It is clear that total number of determinants of second order entries being 0's or 1's is 2^4

Value of determinant is positive for cases as shown

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

i.e favourable case is 3

Hence probability that the value of the determinant is positive is $\frac{3}{16}$.

9.

An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:

$$P(\text{A fails}) = 0.2$$

$$P(\text{B fails alone}) = 0.15$$

$$P(\text{A and B fail}) = 0.15$$

Evaluate the following probabilities

(i) $P(\text{A fails}|\text{B has failed})$

(ii) $P(\text{A fails alone})$

Ans - (i) Let us have the following notations

E: A fails

F: B fails

Given,

$$P(E) = 0.2$$

$$P(E \cap F) = 0.15$$

$$P(E' \cap F) = 0.15$$

We know that

$$P(E' \cap F) = P(E) - P(E \cap F)$$

$$\Rightarrow P(E) = 0.3$$

Now probability that A fails given B has failed is given by

$$P(E|F) = \frac{0.15}{0.3} = 0.5$$

Hence, probability that A fails given B has failed is 0.5.

(ii) Probability that A fails alone is given by

$$P(E \cap F') = P(F) - P(E \cap F)$$

$$P(E \cap F') = 0.05$$

Hence Probability that A fails alone is 0.05

10.

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Ans - Let us have the following notation that

E: the red ball is transferred

$$P(E) = \frac{3}{7}$$

F: the black ball is transferred

$$P(F) = \frac{4}{7}$$

G: the red ball is drawn

When a red ball is transferred

$$P(G|E) = \frac{5}{10}$$

Similarly, when a black ball is transferred

$$P(G|F) = \frac{4}{10}$$

$$P(F|G) = \frac{\frac{4}{7} \times \frac{4}{10}}{\frac{4}{7} \times \frac{4}{10} + \frac{3}{7} \times \frac{5}{10}}$$

Hence probability that the transferred ball is back $\frac{16}{31}$

11.

If A and B are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then

(A) $A \subset B$

(B) $B \subset A$

(C) $B \times \emptyset$

(D) $A = \emptyset$

Ans - Given that, $P(A) \neq 0$ and $P(B|A) = 1$

$$\text{That is } P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow \frac{P(A \cap B)}{P(A)} = 1$$

$$\Rightarrow P(A \cap B) = P(A)$$

$$\therefore A \subset B$$

Hence correct answer is option A.

12.

If $P(A|B) > P(A)$, then which of the following is correct:

(A) $P(B|A) < P(B)$

(C) $P(B|A) > P(B)$

(B) $P(A \cap B) < P(A) \cdot P(B)$

(D) $P(B|A) = P(B)$

Ans - Given that $P(A|B) > P(A)$,

$$\text{That is } \frac{P(A \cap B)}{P(B)} > P(A)$$

$$P(A \cap B) > P(A) \cdot P(B)$$

$$\frac{P(A \cap B)}{P(A)} > P(B)$$

$$P(B|A) > P(B)$$

Hence correct answer is option C.

13.

If A and B are two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then

(A) $P(B|A) = 1$

(C) $P(B|A) = 0$

(B) $P(A|B) = 1$

(D) $P(A|B) = 0$

Ans - Given that, A and B are two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$,

$$P(A) + P(B) - P(A \cap B) = P(A)$$

$$P(B) - P(A \cap B) = 0$$

$$P(A \cap B) = P(B)$$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{1} = 1$$

Hence correct answer is option B.