NCERT Solutions for Class 12 Maths Chapter 12 – Linear Programming

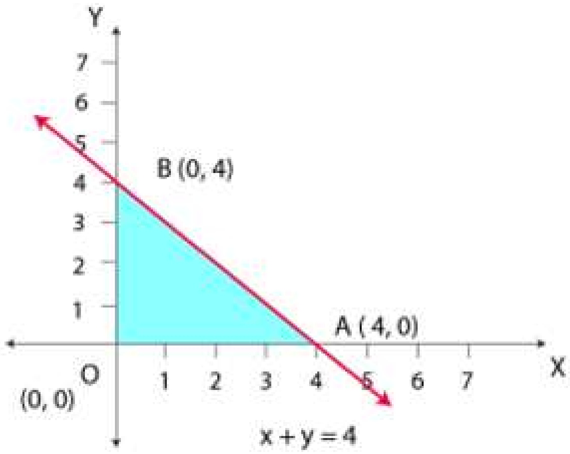
Exercise 12.1

Solve the following Linear Programming Problems graphically:

1.

Maximise $Z = 3x + 4y$ subject to the constraints : $x + y \leq 4$, $x \geq 0, y \geq 0$

Ans - Feasible region determined by constraints, $x + y \le 4$, $x \ge 0$, $y \ge 0$, is given by



Points at the corners in the feasible region are O(0, 0), A(4, 0), and B(0, 4). Values of Z on these points will be,

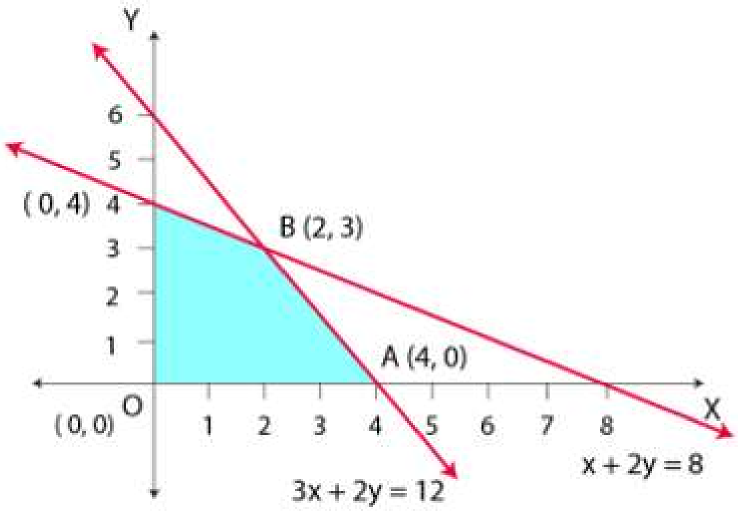
Corner Point	$Z = 3x + 4y$	
O(0,0)	0	
A(4,0)	12	
B(0,4)	16	→ Maximum

Hence, maximum value of Z will be 16 at point B(0, 4).

2.

Minimise $Z = -3x + 4y$ subject to $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$.

Ans - Feasible region determined by constraints, $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$ and $y \geq 0$, is given by



Points at corners in the feasible region are O(0, 0), A(4, 0), B(2, 3) and C(0, 4). Values of Z on these points will be,

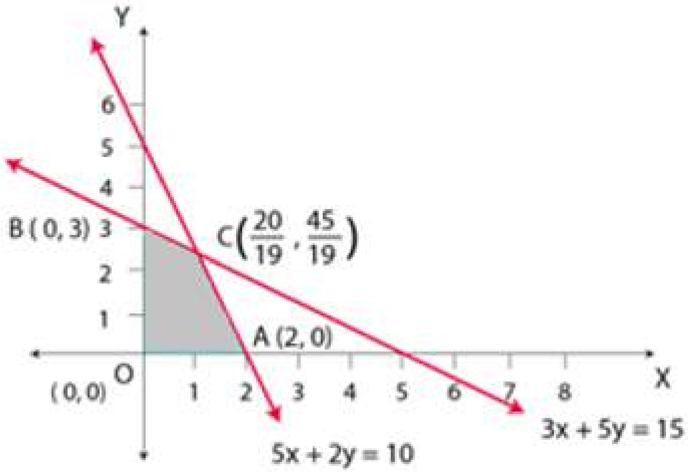
Corner Point	Z=-3x+4y	
O(0,0)	0	
A(4,0)	-12	→ Minimum
B(2,3)	6	
C(4,0)	16	

Hence, minimum value of Z will be -12 at point A(4,0).

3.

Maximise $Z = 5x + 3y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

Ans - Feasible region determined by constraints, $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$ and $y \geq 0$, is given by,



Corner points of the feasible region are O(0, 0), A(2, 0), B(0, 3) and C(20/19, 45/19). Values of Z on these points will be,

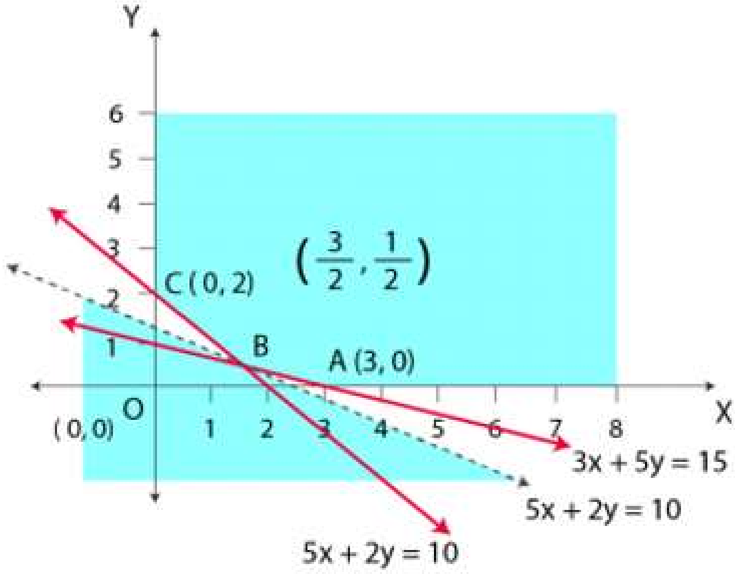
Corner Point	**Z = 5x + 3y**	
O(0,0)	0	
A(2,0)	10	
B(0,3)	9	
C(20/19, 45/19)	235/18	→ Maximum

Hence, maximum value of Z will be 235/18 at point C(20/19, 45/19).

4.

Minimise $Z = 3x + 5y$ such that $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

Ans - Feasible region determined by constraints, $x + 3y \geq 3$, $x + y \geq 2$, and $x, y \geq 0$, is given by,



We can see that feasible region is unbounded.

Corner points of feasible region are A(3, 0), B(3/2, 1/2) and C(0, 2). Values of Z on these points will be,

Corner Point	**Z = 3x + 5y**	
A(3,0)	9	
B(3/2, 1/2)	7	→ Minimum
C(0,2)	10	

Since feasible region is unbounded, we cannot be certain that 7 is the minimum value of Z.

To verify this, we must graph the inequality, $3x + 5y < 7$, and determine if the resulting plane intersects with the feasible region.

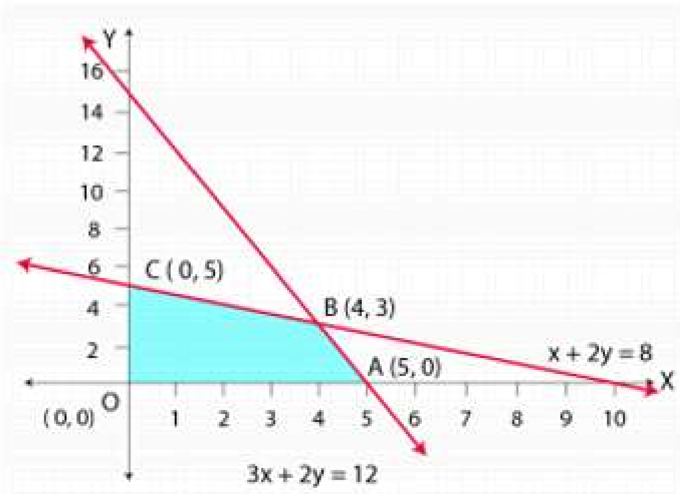
From the graph that we sketched, we can see that there is no common point between feasible regions and the sketched inequality.

Hence, minimum value of Z will be 7 at point B(3/2, 1/2).

5.

Maximise $Z = 3x + 2y$ subject to $x + 2y \leq 10$, $3x + y \leq 15$, x, $y \geq 0$.

Ans - Feasible region determined by constraints, $x + 2y \leq 10$, $3x + y \leq 15$, $x \geq 0$, and $y \geq 0$, is given by,



Corner points of feasible region are A(5, 0), B(4, 3), C(0, 5) and D(0, 0). Values of Z on these points will be,

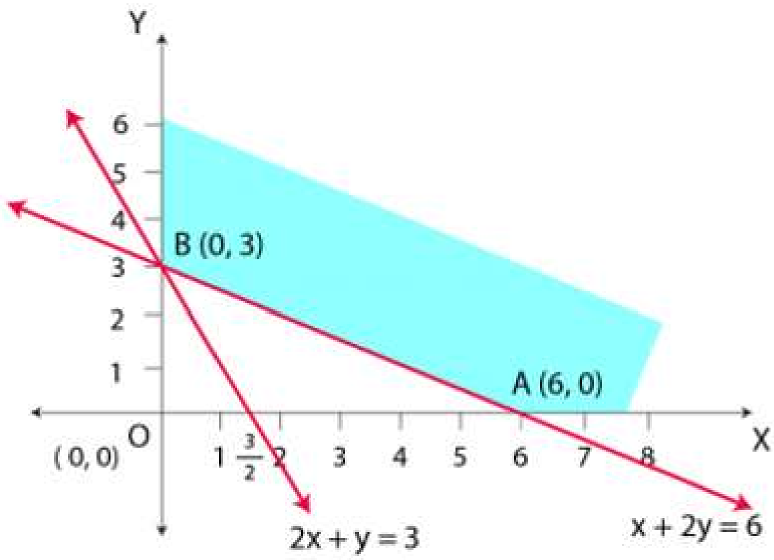
Corner Point	$Z = 3x + 2y$	
A(5,0)	15	
B(4,3)	18	→ Maximum
C(0,5)	10	

Hence, maximum value of Z will be 18 at point B(4, 3).

6.

Minimise $Z = x + 2y$ subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$.

Ans - Feasible region determined by constraints $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$, and $y \geq 0$, is given by,



Corner points of feasible region are A(6, 0) and B(0, 3).
Values of Z on these points will be,

Corner Point	$Z = x + 2y$
A(6, 0)	6
B(0, 3)	6

Value of Z is same at both A and B. Value of Z for any other point in the line like (2, 2) is also 6.

∴ Minimum value of Z occurs at more than two points.

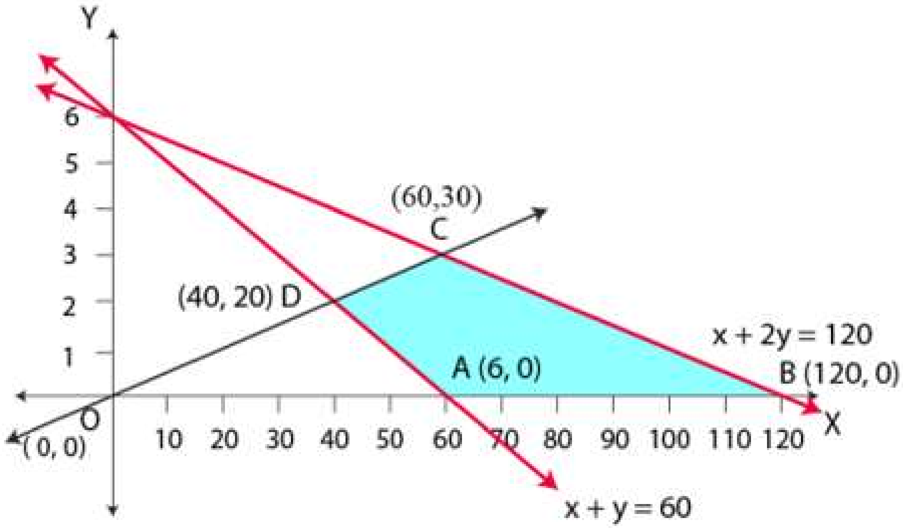
Hence, minimum value of Z will be 6 at at every point on the line $x + 2y = 6$.

Show that the minimum of Z occurs at more than two points

7.

Minimise and Maximise $Z = 5x + 10y$ subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.

Ans - Feasible region determined by constraints $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, and $y \geq 0$, is given by,



Corner points of feasible region are A(60, 0), B(120, 0), C(60, 30), and D(40, 20). Values of Z on these points will be,

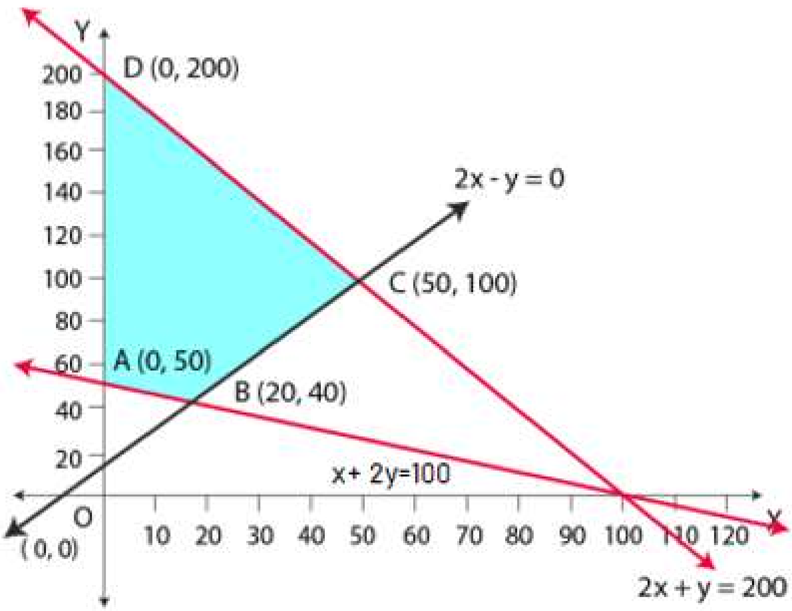
Corner Point	$Z = 5x + 10y$	
A(60, 0)	300	→ Minimum
B(120, 0)	600	→ Maximum
C(60, 30)	600	→ Maximum
D(40,20)	400	

Hence, minimum value of Z will be 300 at point A(60, 0) and maximum value of Z will be 600 at all points on line joining B(120, 0) and C(60, 30).

8.

Minimise and Maximise $Z = x + 2y$ subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$; $x, y \geq 0$.

Ans - Feasible region determined by constraints $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$, is given by,



Corner points of feasible region are A(0, 50), B(20, 40), C(50, 100) and D(0, 200). Values of Z on these points will be,

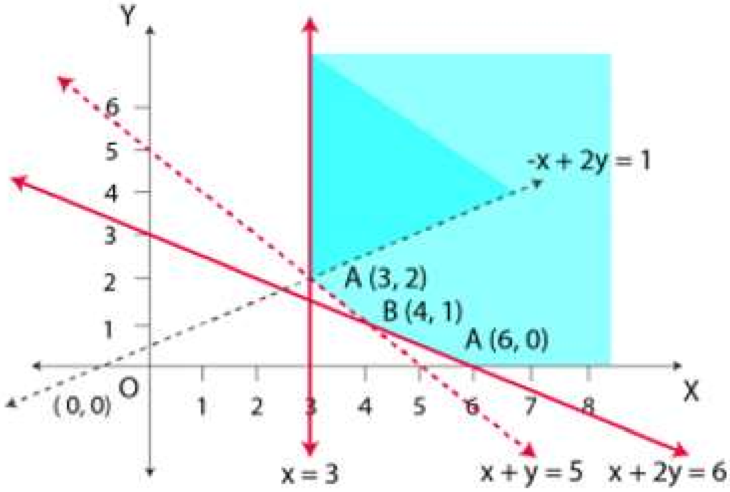
Corner Point	$\mathbf{Z = x + 2y}$	
A(0, 50)	100	→ Minimum
B(20, 40)	100	→ Minimum
C(50, 100)	250	
D(0, 200)	400	→ Maximum

Hence, minimum value of Z will be 100 at all points on line joining A(0, 50) and B(20, 40) and maximum value of Z will be 400 at point D(0, 200).

9.

Maximise $Z = -x + 2y$, subject to the constraints: $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.

Ans - Feasible region determined by constraints x ≥ 3, x + y ≥ 5, x + 2y ≥ 6, and y ≥ 0, is given by,



Here feasible region is unbounded. Corner points of feasible region A(6, 0), B(4, 1) and C(3, 2). Values of Z on these points will be,

Corner Point	**Z = -x + 2y**	
A(6, 0)	Z = -6	
B(4, 1)	Z = -2	
C(3, 2)	Z = 1	→ Maximum

Since feasible region is unbounded, z = 1 may or may not be the maximum value.

For this reason, we graph the inequality - x + 2y > 1 and determine if the resulting half-plane shares points with the viable region. The resulting feasible region shares points with the feasible region.

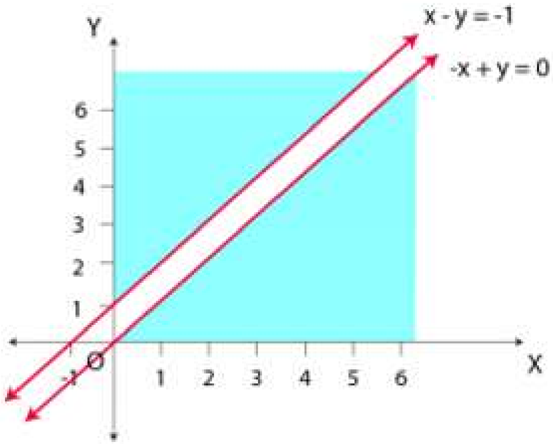
∴ Z = 1 is not the maximum value.

Hence Z has no maximum value.

10.

Maximise Z = x + y, subject to x – y ≤ –1, –x + y ≤ 0, x, y ≥ 0

Ans - Feasible region determined by constraints, $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$, is given by,



There is no feasible region, and hence Z has no maximum value.