

NCERT Solutions for Class 12 Maths

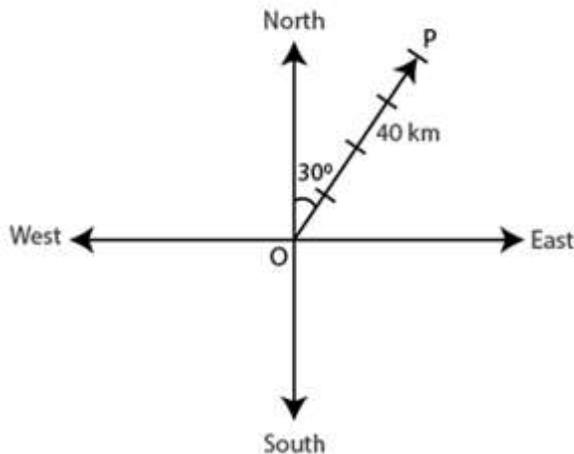
Chapter 10 – Vector Algebra

Exercise 10.1

1.

Represent graphically a displacement of 40 km, 30° east of north.

Ans -



2.

Classify the following measures as scalars and vectors.

- | | |
|------------------------|--------------------------|
| (i) 10 kg | (ii) 2 meters north-west |
| (iii) 40° | (iv) 40 watt |
| (v) 10^{-19} coulomb | (vi) 20 m/s^2 |

Ans - (i) 10 kg is a scalar quantity because it involves only magnitude.

(ii) 2 meters north-west is a vector quantity as it involves both magnitude and direction.

(iii) 40° is a scalar quantity as it involves only magnitude.

(iv) 40 watts is a scalar quantity as it involves only magnitude.

(v) 10^{-19} coulomb is a scalar quantity as it involves only magnitude.

(vi) 20 m/s^2 is a vector quantity as it involves magnitude as well as direction.

3.

Classify the following as scalar and vector quantities.

(i) time period **(ii) distance** **(iii) force**

(iv) velocity **(v) work done**

Ans - (i) Time period is a scalar quantity as it involves only magnitude.

(ii) Distance is a scalar quantity as it involves only magnitude.

(iii) Force is a vector quantity as it involves both magnitude and direction.

(iv) Velocity is a vector quantity as it involves both magnitude as well as direction.

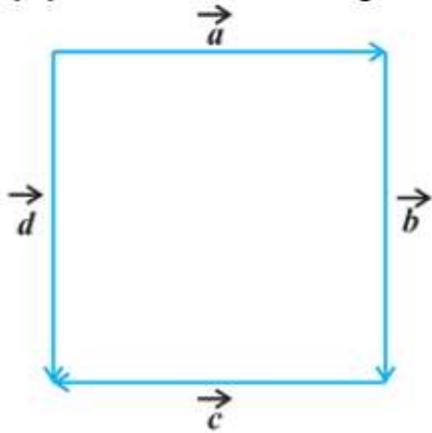
(v) Work done is a scalar quantity as it involves only magnitude.

4.

In Fig (a square), identify the following vectors.

(i) Coinitial (ii) Equal

(iii) Collinear but not equal



Ans - (i) Vectors \vec{a} and \vec{b} are co-initial because they have the same initial point.

(ii) Vectors \vec{b} and \vec{d} are equal because they have the same magnitude and direction.

(iii) Vectors \vec{a} and \vec{c} are collinear but not equal. This is because although they are parallel, their directions are not the same.

5.

Answer the following as true or false.

(i) \vec{a} and $-\vec{a}$ are collinear.

(ii) Two collinear vectors are always equal in magnitude.

(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.

Ans - (i) True. Vectors \vec{a} and $-\vec{a}$ can be parallel or coinciding vectors. In either case, the vectors will have the same magnitude but opposing directions and be parallel to the same line.

(ii) False. Collinear vectors are those vectors that are parallel to the same line.

(iii) False. Two vectors of equal magnitude do not have to be parallel to the same line.

(iv) False. Two vectors are said to be equal if they have the same magnitude and direction, independent of the positions of their starting points.

Exercise 10.2

1.

Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k};$$

$$\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k};$$

$$\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Ans: Magnitude of a vector say, $\vec{a}_1 = \hat{i}_1 + \hat{j}_1 + \hat{k}_1$ is,

$$|\vec{a}_1| = \sqrt{(i_1)^2 + (j_1)^2 + (k_1)^2}$$

$$\Rightarrow |\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

For \vec{b} ,

$$|\vec{b}| = \sqrt{(2)^2 + (-7)^2 + (-3)^2} = \sqrt{4 + 49 + 9} = \sqrt{62}$$

For \vec{c} ,

$$|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right)^2} = 1$$

2.

Write two different vectors having same magnitude.

Ans - Let us assume 2 vectors \vec{a} and \vec{b}

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$$

Magnitude of a vector say, $\vec{a}_1 = \hat{i}_1 + \hat{j}_1 + \hat{k}_1$ is.,

$$|\vec{a}_1| = \sqrt{(i_1)^2 + (j_1)^2 + (k_1)^2}$$

$$\Rightarrow |\vec{a}| = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

Now for \vec{b} ,

$$|\vec{b}| = \sqrt{(2)^2 + (-1)^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$$

3.

Write two different vectors having same direction.

Ans - Let us assume the two vectors as,

$$\vec{p} = \hat{i} + \hat{j} + \hat{k} \text{ and } \vec{q} = 2\hat{i} + 2\hat{j} + 2\hat{k}$$

Direction cosines of the two vectors can be calculated as,

For \vec{p} ,

$$l = \frac{1}{\sqrt{(1)^2 + (1)^2 + (1)^2}} = \frac{1}{\sqrt{3}}$$

$$m = \frac{1}{\sqrt{(1)^2 + (1)^2 + (1)^2}} = \frac{1}{\sqrt{3}}$$

$$n = \frac{1}{\sqrt{(1)^2 + (1)^2 + (1)^2}} = \frac{1}{\sqrt{3}}$$

For \vec{q} ,

$$l = \frac{2}{\sqrt{(2)^2 + (2)^2 + (2)^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$m = \frac{2}{\sqrt{(2)^2 + (2)^2 + (2)^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$n = \frac{2}{\sqrt{(2)^2 + (2)^2 + (2)^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

It can be clearly observed that direction cosines obtained in case of both the vectors \vec{p} and \vec{q} are same and therefore they have got same direction but different vectors.

4.

Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.

Ans - Observe that it is required for the corresponding coefficients of \hat{i} and \hat{j} to be equal in order to have two equal vectors.

$$\Rightarrow x = 2, y = 3$$

5.

Find the scalar and vector components of the vector with initial point $(2, 1)$ and terminal point $(-5, 7)$.

Ans - Given initial and terminal point are $(2, 1)$ and $(-5, 7)$.

$$\Rightarrow \vec{PQ} = (-5 - 2)\hat{i} + (7 - 1)\hat{j} = -7\hat{i} + 6\hat{j}$$

6.

Find the sum of the vectors with initial point $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

Ans - Sum of the vectors can be calculated as,

$$\vec{a} + \vec{b} + \vec{c} = \hat{i} - 2\hat{j} + \hat{k} + (-2\hat{i}) + 4\hat{j} + 5\hat{k} + \hat{i} - 6\hat{j} - 7\hat{k}$$

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} = (1 - 2 + 1)\hat{i} + (-2 + 4 - 6)\hat{j} \\ + (1 + 5 - 7)\hat{k}$$

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} = -4\hat{j} - \hat{k}$$

7.

Ans - Unit vector in the direction of $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is represented as \hat{a} and can be calculated as,

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (2)^2} = \sqrt{6}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

8.

Find the unit vector in the direction of the vector, \overrightarrow{PQ} , where P and Q are the points (1, 2, 3) and (4, 5, 6) respectively.

Ans - Given points P - (1, 2, 3) and Q - (4, 5, 6).

$$\overrightarrow{PQ} = (4 - 1)\hat{i} + (5 - 2)\hat{j} + (6 - 3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

Unit vector in the direction of $\overrightarrow{PQ} = 3\hat{i} + 3\hat{j} + 3\hat{k}$ is represented as \widehat{PQ} and can be calculated as

$$|\overrightarrow{PQ}| = \sqrt{(3)^2 + (3)^2 + (3)^2} = \sqrt{27} = 3\sqrt{3}$$

$$\therefore \widehat{PQ} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

9.

For the given vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$ find the unit vector in the direction of the vector, $\vec{a} + \vec{b}$.

Ans - Given vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$.

$$\vec{a} + \vec{b} = (2 - 1)\hat{i} + (-1 + 1)\hat{j} + (2 + 1)\hat{k} = \hat{i} + 3\hat{k}$$

Unit vector in the direction of $\vec{a} + \vec{b} = \hat{i} + 3\hat{k}$ is represented as $\widehat{a + b}$ and can be calculated as

$$|\vec{a} + \vec{b}| = \sqrt{(1)^2 + (0)^2 + (3)^2} = \sqrt{10} = \sqrt{10}$$

$$\therefore \widehat{a + b} = \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{\hat{i} + 3\hat{k}}{\sqrt{10}} = \frac{1}{\sqrt{10}}\hat{i} + \frac{3}{\sqrt{10}}\hat{k}$$

10.

Find a vector in the direction of the vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

Ans - Assume the given vector as, $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$

Unit vector in the direction of $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$ is represented as \hat{a} and can be calculated as

$$|\vec{a}| = \sqrt{(5)^2 + (-1)^2 + (2)^2} = \sqrt{30}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}} = \frac{5}{\sqrt{30}}\hat{i} - \frac{1}{\sqrt{30}}\hat{j} + \frac{2}{\sqrt{30}}\hat{k}$$

Now, a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units can be calculated as,

$$8\hat{a} = 8 \left(\frac{5}{\sqrt{30}}\hat{i} - \frac{1}{\sqrt{30}}\hat{j} + \frac{2}{\sqrt{30}}\hat{k} \right) = \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

11.

Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Ans - Assume given vector as, $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

Therefore, it can be observed that \vec{b} can be deduced in terms of \vec{a} as.

$$\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$$

$$\therefore \vec{b} = \lambda\vec{a}$$

Value of $\lambda = -2$ in this case and therefore the given two vectors collinear.

12.

Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

Ans - Assume the given vector as, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\Rightarrow |\vec{a}| = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{14}$$

Direction cosines of the vector can be calculated as,

$$\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right)$$

13.

Find the direction cosines of the vector joining the points A (1, 2, -3) and B (-1, -2, 1), directed from A to B.

Ans - Given points are A (1, 2, -3) and B(-1, -2, 1)

$$\begin{aligned} \therefore \vec{AB} &= (-1 - 1)\hat{i} + (-2 - 2)\hat{j} + (1 - (-3))\hat{k} \\ &= -2\hat{i} - 4\hat{j} + 4\hat{k} \end{aligned}$$

$$\Rightarrow |\vec{AB}| = \sqrt{(-2)^2 + (-4)^2 + (4)^2} = 6$$

Hence, direction cosines of the vector can be calculated as,

$$\left(\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3} \right)$$

14.

Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY and OZ .

Ans - Consider the given vector as $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$\Rightarrow |\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

Direction cosines of the vector can be calculated as,

$$\therefore \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

Assume, α, β and γ to be the angles formed by \vec{a} with the axes OX, OY and OZ .

$$\therefore \cos\alpha = \frac{1}{\sqrt{3}}, \cos\beta = \frac{1}{\sqrt{3}}, \cos\gamma = \frac{1}{\sqrt{3}}$$

This shows that the given vector is equally inclined to the the positive direction of all the three axes.

15.

Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio 2 : 1

(i) internally (ii) externally

Ans - (i) Observe that given position vectors of the two points, P and Q are, $\vec{OP} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{OQ} = -\hat{i} + \hat{j} + \hat{k}$

Therefore, the position vector of R dividing the line joining the two given points through an internal division in the ratio of 2:1 can be calculated using the formula, $\frac{m\vec{b} + n\vec{a}}{m+n}$ (where the ratio is represented by $m:n$), as shown below

$$\begin{aligned} \vec{OR} &= \frac{2(-\hat{i} + \hat{j} + \hat{k}) + 1(\hat{i} + 2\hat{j} - \hat{k})}{2 + 1} = \frac{-\hat{i} + 4\hat{j} + \hat{k}}{3} \\ &= \frac{-1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k} \end{aligned}$$

(ii) Observe that given position vectors of the two points, P and Q are, $\vec{OP} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{OQ} = -\hat{i} + \hat{j} + \hat{k}$

Therefore, the position vector of R dividing the line joining the two given points through an external division in the ratio of 2:1 can be calculated using the formula, $\frac{m\vec{b} - n\vec{a}}{m - n}$ (where the ratio is represented by $m:n$) as shown below

$$\vec{OR} = \frac{2(-\hat{i} + \hat{j} + \hat{k}) - 1(\hat{i} + 2\hat{j} - \hat{k})}{2 - 1} = \frac{-3\hat{i} - \hat{k}}{1} = -3\hat{i} + 3\hat{k}$$

16.

Find the position vector of the mid point of the vector joining the points $P(2, 3, 4)$ and $Q(4, 1, -2)$.

Ans - Consider midpoint of vector joining the position vectors of the two points, $\vec{OP} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{OQ} = 4\hat{i} + \hat{j} - 2\hat{k}$ to be R .

Therefore, position vector of R dividing the line joining two given points through an internal division in the ratio of 1:1 can be calculated as,

$$\begin{aligned}\vec{OR} &= \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{1 + 1} = \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} \\ &= 3\hat{i} + 2\hat{j} + \hat{k}\end{aligned}$$

17.

Show that the positions A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively form the vertices of a right-angled triangle.

Ans - Position vectors of points A, B and C are,

$$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

Now, AB, BC and CA can be calculated as.

$$\begin{aligned}\overrightarrow{AB} &= \vec{b} - \vec{a} = (2 - 3)\hat{i} + (-1 + 4)\hat{j} + (1 + 4)\hat{k} \\ &= -\hat{i} + 3\hat{j} + 5\hat{k}\end{aligned}$$

$$\begin{aligned}\overrightarrow{BC} &= \vec{c} - \vec{b} = (1 - 2)\hat{i} + (-3 + 1)\hat{j} + (-5 - 1)\hat{k} \\ &= -\hat{i} - 2\hat{j} - 6\hat{k}\end{aligned}$$

$$\begin{aligned}\overrightarrow{CA} &= \vec{a} - \vec{c} = (3 - 1)\hat{i} + (-4 + 3)\hat{j} + (-4 + 5)\hat{k} \\ &= 2\hat{i} - \hat{j} + \hat{k}\end{aligned}$$

Calculating squares of magnitudes we get,

$$|AB|^2 = (-1)^2 + (3)^2 + (5)^2 = 35$$

$$|BC|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 41$$

$$|CA|^2 = (2)^2 + (-1)^2 + (1)^2 = 6$$

Using Pythagoras theorem it can be shown that this triangle is right-angled triangle as shown below,

$$|AB|^2 + |CA|^2 = 35 + 6 = 41 = |BC|^2$$

18.

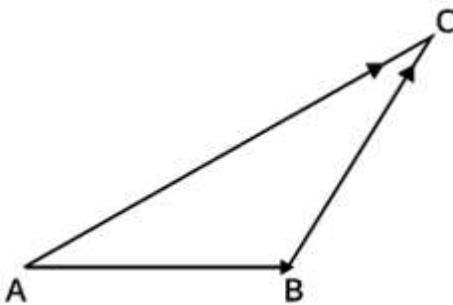
In triangle ABC which of the following is not true:

(A) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

(B) $\overrightarrow{AB} + \overrightarrow{BC} \cdot \overrightarrow{AC} = \vec{0}$

(C) $\overrightarrow{AB} + \overrightarrow{BC} \cdot \overrightarrow{CA} = \vec{0}$

(D) $\overrightarrow{AB} \cdot \overrightarrow{CB} + \overrightarrow{CB}$



Ans - Considering triangle law of addition we can write,

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$$

∴ Equation in (A) is true.

Again,

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$$

∴ Equation in (B) is true.

Again,

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$$

∴ Equation in (D) is true.

Now, for the equation in (C),

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} \neq \vec{0}$$

Hence, the equation in (C) is incorrect. Thus option C is the correct answer.

19.

If \vec{a} and \vec{b} are two collinear vectors, then which of the following are incorrect.

(A) $\vec{b} = \lambda\vec{a}$, for some scalar λ .

(B) $\vec{a} = \pm\vec{b}$

(C) the respective components of \vec{a} and \vec{b} are not proportional.

(D) both the vectors \vec{a} and \vec{b} have same direction, but different magnitudes.

Ans - If \vec{a} and \vec{b} are collinear vectors then they are parallel.

$\vec{b} = \lambda \vec{a}$, for some scalar λ .

Again, if $\lambda = \pm 1$, then $\vec{a} = \pm\vec{b}$

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then,

$\vec{b} = \lambda\vec{a}$

$b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$

$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda a_1\hat{i} + \lambda a_2\hat{j} + \lambda a_3\hat{k}$

$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$

$\Rightarrow \lambda = \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3}$

This shows that components of \vec{a} and \vec{b} , are proportional but the vectors can have different directions.

Hence, the statement in (D) is incorrect. Thus option D is the correct answer.

Exercise 10.3

1.

Find the angle between two vectors \vec{a} and \vec{b} with magnitude $\sqrt{3}$ and 2, respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$

Ans - Given $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = 2$

$$\vec{a} \cdot \vec{b} = \sqrt{6}$$

Also, we know that

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$, where θ is the angle between vectors.

$$\Rightarrow \sqrt{6} = 2\sqrt{3} \cos \theta$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, the angle between the vectors is $\frac{\pi}{4}$.

2.

Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

Ans - Let the given vectors be

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$$

We know that

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$, where θ is the angle between the vectors

$$\begin{aligned} \Rightarrow (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} - 2\hat{j} + \hat{k}) \\ = \sqrt{1^2 + 2^2 + 3^2} \sqrt{3^2 + 2^2 + 1^2} \end{aligned}$$

$$\Rightarrow 10 = 14 \cos \theta$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{5}{7} \right)$$

Hence the angle between the vectors is $\theta = \cos^{-1} \left(\frac{5}{7} \right)$

3.

Find the projection of the vector $\vec{i} - \vec{j}$ on the vector $\vec{i} + \vec{j}$

Ans - Given we have vectors

$$\vec{a} = \hat{i} - \hat{j} \text{ and}$$

$$\vec{b} = \hat{i} + \hat{j}$$

Now the projection of \vec{a} on \vec{b} is given by

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{(\hat{i} - \hat{j}) \cdot (\hat{i} + \hat{j})}{\sqrt{2}}$$

Hence, the projection of $\hat{i} - \hat{j}$ on $\hat{i} + \hat{j}$ is 0.

4.

Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

Ans - Given we have vectors.

$$\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k} \text{ and}$$

$$\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$$

Now the projection of \vec{a} on \vec{b} is given by

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{(\hat{i} + 3\hat{j} + 7\hat{k}) \cdot (7\hat{i} - \hat{j} + 8\hat{k})}{\sqrt{114}}$$

Hence, the projection of $\hat{i} - \hat{j}$ on $\hat{i} + \hat{j}$ is $\frac{60}{\sqrt{114}}$.

5.

Show that each of the given three vectors is a unit vector:

$$\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}), \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$$

Also, show that they are mutually perpendicular to each other.

Ans - Let us have following notations

$$7\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$7\vec{b} = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

$$7\vec{c} = 6\hat{i} + 2\hat{j} - 3\hat{k}$$

Its magnitude is given by

$$7a = \sqrt{2^2 + 3^2 + 6^2}$$

$$\Rightarrow a = \frac{7}{7} = 1$$

Hence, the magnitude is 1, it is a unit vector

Similarly, magnitude of $7\vec{b}$ is given by

$$7b = \sqrt{3^2 + 6^2 + 2^2}$$

$$\Rightarrow b = \frac{7}{7} = 1$$

Hence, the magnitude is 1, it is a unit vector.

Similarly, magnitude of $7\vec{c}$ is given by

$$7c = \sqrt{6^2 + 2^2 + 3^2}$$

$$\Rightarrow c = \frac{7}{7} = 1$$

Hence, the magnitude is 1, it is a unit vector

Now we will calculate the followings

$$\vec{a} \cdot \vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 0$$

$$\vec{c} \cdot \vec{b} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 0$$

$$\vec{a} \cdot \vec{c} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot (6\hat{i} + 2\hat{j} - 3\hat{k}) = 0$$

Hence, all these vectors are mutually perpendicular.

6.

Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$

Ans - Given $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8 \dots \dots (1)$

$$|\vec{a}| = 8|\vec{b}| \quad \dots \dots (2)$$

Now from (1)

$$|\vec{a}|^2 - |\vec{b}|^2 = 8$$

Also from (2)

$$|\vec{b}|^2 (64 - 1) = 8$$

$$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}}$$

$$\therefore |\vec{a}| = 8 \sqrt{\frac{8}{63}}$$

7.

Evaluate the product $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$

Ans - Given two vectors $3\vec{a} - 5\vec{b}$ and $2\vec{a} + 7\vec{b}$

$$(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}) = 6a^2 + 11\vec{a} \cdot \vec{b} - 35b^2$$

8.

Find the magnitude of two vectors \vec{a} and \vec{b} , having the same magnitude and such that the angle between them is 60° and their scalar product is $\frac{1}{2}$.

Ans - Given $\theta = 60^\circ$

$$\vec{a} \cdot \vec{b} = \frac{1}{2}$$

According to the question

$$\frac{1}{2} = |\vec{a}||\vec{b}| \cos 60$$

$$\Rightarrow \frac{1}{2} = |\vec{a}||\vec{b}| \frac{1}{2}$$

$$\Rightarrow |\vec{a}| = |\vec{b}| = 1$$

9.

Find $|\vec{x}|$, if for a unit vector \vec{a} , $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$

Ans - Given $|\vec{a}| = 1$

$$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$$

$$\Rightarrow |\vec{x}|^2 - 1 = 12 \text{ (since } |\vec{a}| = 1)$$

$$\Rightarrow |\vec{x}| = \sqrt{13}$$

10.

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ perpendicular to \vec{c} , then find the value of λ .

Ans - Given vectors are

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\therefore \vec{a} + \lambda\vec{b} = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 - \lambda)\hat{k}$$

$$\vec{c} = 3\hat{i} + \hat{j}$$

According to the question

$$(\vec{a} + \lambda\vec{b}) \cdot (3\hat{i} + \hat{j}) = 0$$

$$\Rightarrow \left((2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 - \lambda)\hat{k} \right) \cdot (3\hat{i} + \hat{j}) = 0$$

$$\Rightarrow (2 - \lambda) \cdot 3 + (2 + 2\lambda) \cdot 1 + (3 - \lambda) \cdot 0 = 0$$

$$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow 8 - \lambda = 0$$

$$\Rightarrow \lambda = 8$$

Hence we get $\lambda = 8$.

11.

Show that: $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two nonzero vectors \vec{a} and \vec{b} .

Ans - Let us suppose the two vectors as shown.

$$\vec{\eta} = |\vec{a}|\vec{b} + |\vec{b}|\vec{a}$$

$$\vec{\kappa} = |\vec{a}|\vec{b} - |\vec{b}|\vec{a}$$

$$\text{Now } \vec{\eta} \cdot \vec{\kappa} = (|\vec{a}|\vec{b} + |\vec{b}|\vec{a}) \cdot (|\vec{a}|\vec{b} - |\vec{b}|\vec{a})$$

$$\Rightarrow \vec{\eta} \cdot \vec{\kappa} = |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2 = 0$$

Hence proved.

12.

If $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded above the vector \vec{b} ?

Ans - Given two equations

$$\vec{a} \cdot \vec{a} = 0 \quad \dots (1)$$

$$\vec{a} \cdot \vec{b} = 0$$

It is clear from the equation (1) that $|\vec{a}|^2 = 0$

Hence, \vec{b} can be any vector

13.

If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

Ans - Given we have three unit vectors, $\vec{a}, \vec{b}, \vec{c}$ such that

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\Rightarrow (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = \frac{-3}{2}$$

14.

If either $\vec{a} = 0$ or $\vec{b} = 0$ vector, then. But, the converse need not be true. Justify your answer with an example.

Ans - Let us suppose two vectors as shown

$$\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$$

$$\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$$

$$\text{Now } \vec{a} \cdot \vec{b} = (2\hat{i} + 4\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 3\hat{j} - 6\hat{k})$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 6 + 12 - 18 = 0$$

But clearly neither $\vec{a} = 0$ nor $\vec{b} = 0$

15.

If the vertices A, B, C of a triangle ABC are (1,2,3), (-1,0,0) and (0,1,2) respectively then find $\angle ABC$. [$\angle ABC$ is the angle between the vectors \vec{BA} and \vec{BC}]

Ans - Given that the vertices of $\triangle ABC$ are

$$A(1,2,3)$$

$$B(-1,0,0)$$

$$C(0,1,2)$$

Now $\angle ABC$ is the angle between \vec{BA} and \vec{BC}

$$\therefore \vec{BA} \cdot \vec{BC} = (2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k})$$

$$\Rightarrow 10 = \sqrt{17}\sqrt{6} \cos \angle ABC$$

$$\text{Hence, we found that } \angle ABC = \cos\left(\frac{10}{\sqrt{17}\sqrt{6}}\right)$$

16.

Show that the points A (1,2,7), B(2, 6, 3) and C(3, 10, -1) are collinear

Ans - Given we have points as shown

$$A(1,2,7)$$

$$B(2,6,3)$$

$$C(3,10,-1)$$

$$\overrightarrow{AB} = (2 - 1)\hat{i} + (6 - 2)\hat{j} + (3 - 7)\hat{k}$$

$$\Rightarrow \overrightarrow{AB} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overrightarrow{BC} = (3 - 2)\hat{i} + (10 - 6)\hat{j} + (-1 - 3)\hat{k}$$

$$\Rightarrow \overrightarrow{BC} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overrightarrow{AC} = (3 - 1)\hat{i} + (10 - 2)\hat{j} + (-1 - 7)\hat{k}$$

$$\Rightarrow \overrightarrow{AC} = 2\hat{i} + 8\hat{j} - 8\hat{k}$$

$$\Rightarrow \overrightarrow{AC} = 2(\hat{i} + 4\hat{j} - 4\hat{k})$$

Clearly,

$$|\overrightarrow{AC}| = |\overrightarrow{BC}| + |\overrightarrow{AB}|$$

Hence, vectors are collinear.

17.

Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$, $3\hat{i} - 4\hat{j} - 4\hat{k}$ forms the vertices of the right angled triangle.

Ans - Given that

$$\vec{OA} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\vec{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

Where $\vec{OA}, \vec{OB}, \vec{OC}$ are position vectors of A, B, C

$$\vec{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$|\vec{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{41}$$

$$\vec{BC} = 2\hat{i} - \hat{j} + \hat{k}$$

$$|\vec{BC}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$

$$\vec{AC} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$|\vec{AC}| = \sqrt{1^2 + (-3)^2 + (-5)^2} = \sqrt{35}$$

$$\text{Clearly } |\vec{AC}| + |\vec{BC}| = \sqrt{41} = |\vec{AB}|$$

18.

If \vec{a} is a non-zero vector of magnitude 'a' and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if

- (A) $\lambda = 1$ (B) $\lambda = -1$
(C) $a = |\lambda|$ (D) $a = 1/|\lambda|$

Ans - Given we have a vector \vec{a} and a scalar λ

For $\lambda\vec{a}$ to be a unit vector

$$|\lambda\vec{a}| = 1$$

$$a = \frac{1}{|\lambda|}$$

Hence option D is the correct answer.

Exercise 10.4

1.

Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

Ans - Given vectors are

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$$

$$\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

Then the cross-product between the vectors is given by

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(-14 + 14) + -\hat{j}(2 - 21) + \hat{k}(-2 + 21)$$

$$= 19\hat{j} + 19\hat{k}.$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{19^2 + 19^2}$$

$$= \sqrt{2 \times 19^2}$$

$$= 19\sqrt{2}$$

2.

Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Ans - Given vectors are

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

Adding and subtracting vectors successively, we get

$$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$$

$$\vec{a} - \vec{b} = 2\hat{i} + 4\hat{j}$$

$$\Rightarrow (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$$= 16\hat{i} - 16\hat{j} - 8\hat{k}.$$

Then, its magnitude will be,

$$\begin{aligned} |(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| &= \sqrt{16^2 + (-16)^2 + (-8)^2} \\ &= \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2} \\ &= 8\sqrt{2^2 + 2^2 + 1} \\ &= 8\sqrt{9} \\ &= 8 \times 3 \\ &= 24 \end{aligned}$$

Thus, unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is provided as,

$$\begin{aligned} &= \pm \frac{(\vec{a} + \vec{b})(\vec{a} - \vec{b})}{|(\vec{a} + \vec{b})(\vec{a} - \vec{b})|} \\ &= \pm \frac{16i - 16j - 8k}{24} \\ &= \pm \frac{2\hat{i} - 2\hat{j} - \hat{k}}{3} \\ &= \pm \frac{2}{3}\hat{i} \mp \frac{2}{3}\hat{j} \mp \frac{1}{3}\hat{k} \end{aligned}$$

3.

If a unit vector \vec{a} makes angles $\frac{\pi}{3}$ with \vec{i} , $\frac{\pi}{4}$ with \vec{j} and an acute angle θ with \vec{k} , then find θ and hence, the components of \vec{a} .

Ans - Suppose the components of the given unit vector \vec{a} is (a_1, a_2, a_3)

$$\text{Then, } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

Also, $|\vec{a}| = 1$ as \vec{a} is a unit vector

Again, we are provided that, the vector \vec{a} makes angles $\frac{\pi}{3}$ with \hat{i} , $\frac{\pi}{4}$ with \hat{j} and an acute angle θ with \hat{k}

$$\Rightarrow \cos \frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{2} = a_1, \text{ since } \vec{a} \text{ is a unit vector}$$

$$\text{Also, } \cos \frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = a_2, \text{ since } \vec{a} \text{ is a unit vector}$$

$$\text{Now, let } \cos \theta = \frac{a_3}{|\vec{a}|}$$

$$a_3 = \cos \theta, \text{ since, } |\vec{a}| = 1$$

$$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 \theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

$$\text{Thus, } a_3 = \cos \frac{\pi}{3} = \frac{1}{2}$$

$\therefore \theta = \frac{\pi}{3}$ and the components of vector \vec{a} are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

4.

Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

Ans - Given cross-product can be written as

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$$

$$= (\vec{a} - \vec{b}) \times \vec{a} + (\vec{a} - \vec{b}) \times \vec{b} \text{ (using the distributive property of vector product over addition)}$$

$$\vec{a} \times \vec{a} - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{b} \text{ (Distributive property of vector product over addition)}$$

5.

Find λ and μ if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k})$

Ans - Given vector equation can be written as,

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\Rightarrow i(6\mu - 27\lambda) - j(2\mu - 27) + k(2\lambda - 6) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

Comparing scalar components on both sides of the equation we get,

$$6\mu - 27\lambda = 0 \dots (i)$$

$$2\mu - 27 = 0 \dots (ii)$$

$$2\lambda - 6 = 0 \dots (iii)$$

Hence, on solving the equations (i), (ii) and (iii), we get

$$\lambda = 3 \text{ and } \mu = \frac{27}{2}$$

6.

Given that $\vec{a} \cdot \vec{b} = 0$ **and** $\vec{a} \times \vec{b} = \vec{0}$. **What can you conclude about the vector \vec{a} and \vec{b} ?**

Ans - Since, $\vec{a} \cdot \vec{b} = 0$, then we can say that either $|\vec{a}| = 0$, or $|\vec{b}| = 0$ or the vectors \vec{a} and \vec{b} are perpendicular to each other.

Again, since $\vec{a} \times \vec{b} = 0$, it can be said that either $|\vec{a}| = 0$, or $|\vec{b}| = 0$, or the vectors \vec{a} and \vec{b} are parallel to each other.

But, since the vectors cannot be perpendicular and parallel at simultaneous, so the only possibility is either $|\vec{a}| = 0$ or $|\vec{b}| = 0$

7.

Let the vectors $\vec{a}, \vec{b}, \vec{c}$ given as $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then show that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

Ans - Given vectors are

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k},$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$\text{So, } (\vec{b} + \vec{c}) = (b_1 + c_1)\hat{i} + (b_2 + c_2)\hat{j} + (b_3 + c_3)\hat{k}$$

$$\Rightarrow \vec{a} \times (\vec{b} + \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$$

$$= \hat{i}[a_2(b_3 + c_3) - a_3(b_2 + c_2)] \\ - \hat{j}[a_1(b_3 + c_3) - a_3(b_1 + c_1)]$$

$$+ \hat{k}[a_1(b_2 + c_2) - a_2(b_1 + c_1)]$$

$$= \hat{i}(a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2) \\ + \hat{j}(-a_1b_3 - a_1c_3 + a_3b_1 + a_3c_1)$$

$$+ \hat{k}(a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1) \quad \dots (i)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \hat{i}(a_2b_3 - a_3b_2) + \hat{j}(a_3b_1 - a_1b_3) + \hat{k}(a_1b_2 - a_2b_1) \quad \dots (ii)$$

Also,

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \hat{i}(a_2c_3 - a_3c_2) + \hat{j}(a_3c_1 - a_1c_3) + \hat{k}(a_1c_2 - a_2c_1) \quad \dots (iii)$$

Now, add the equations (ii) and (iii). Then, it yields

$$(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$$

$$= \hat{i}(a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2) + \hat{j}(b_1a_3 + a_3c_1 - a_1b_3 - a_1c_3) + \hat{k}(a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1) \quad \dots (iv)$$

Then, the equations (i) and (iv) together implies that

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

Hence proved.

8.

If either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \times \vec{b} = \vec{0}$. Is the converse true? Justify your answer with an example

Ans - Consider any parallel nonzero vectors so that

$$\vec{a} \times \vec{b} = \vec{0}$$

Let nonzero vectors be

$$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\Rightarrow \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix}$$

$$= \hat{i}(24 - 24) - \hat{j}(16 - 16) + \hat{k}(12 - 12)$$

$$= 0\hat{i} + 0\hat{j} + 0\hat{k}$$

Magnitudes of the vectors are given by

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}.$$

Thus, observing the above results, it is found that $\vec{a} \times \vec{b} = \vec{0}$,

while $\vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{0}$. Therefore, it is justified that the converse of the given statement need not be true.

9.

Find the area of the triangle with vertices A (1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

Ans - ΔABC has vertices A (1,1,2), B (2,3,5) and C (1,5,5).

Adjacent sides of triangle ΔABC are \overrightarrow{AB} and \overrightarrow{BC} such that

$$\overrightarrow{AB} = (2 - 1)\hat{i} + (3 - 1)\hat{j} + (5 - 2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overrightarrow{BC} = (1 - 2)\hat{i} + (5 - 3)\hat{j} + (5 - 5)\hat{k} = -\hat{i} + 2\hat{j}.$$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix}$$

$$= \hat{i}(-6) - \hat{j}(3) + \hat{k}(2 + 2)$$

$$= -6\hat{i} - 3\hat{j} + 4\hat{k}.$$

$$|\vec{AB} \times \vec{BC}| = \sqrt{(-6)^2 + (-3)^2 + 4^2}$$

$$= \sqrt{36 + 9 + 16} = \sqrt{61}.$$

⇒ Area of the triangle ΔABC

$$= \frac{1}{2} \times |\vec{AB} \times \vec{BC}|$$

$$= \frac{1}{2} \times \sqrt{61}$$

$$= \frac{\sqrt{61}}{2} \text{ sq. units}$$

10.

Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

Ans - Let given vectors are

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$$

$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$ such that they are the adjacent sides of the parallelogram.

Cross-product between the vectors will be

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= (-1 + 21)\hat{i} - (1 - 6)\hat{j} + (-7 + 2)\hat{k}$$

$$= 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + 5^2}$$

$$= \sqrt{400 + 25 + 25}$$

$$= 15\sqrt{2}$$

Thus, the area of the parallelogram is $15\sqrt{2}$ square units.

11.

Let the vector \vec{a} and \vec{b} be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is unit vector, if the angle between \vec{a} and \vec{b} is

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Ans - Given $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$

We know that $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta \cdot \hat{n}$ where \hat{n} is a unit vector that is perpendicular to both the vectors \vec{a} and \vec{b} ; θ is the angle the vectors \vec{a} and \vec{b} .

Since, $\vec{a} \times \vec{b}$ is unit vector,

$$\Rightarrow |\vec{a} \times \vec{b}| = 1$$

$$\Rightarrow \left| |\vec{a}||\vec{b}|\sin\theta\hat{n} \right| = 1$$

$$\Rightarrow 3 \times \frac{\sqrt{2}}{3} \times \sin\theta = 1$$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, the correct answer is option B.

12.

Area of a rectangle having vertices A, B, C, and D with position vectors $-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ and $-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ respectively is

- (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4

Ans - Position vectors of the vertices A, B, C, D of the rectangle ABCD are such that

$$\vec{OA} = -\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k},$$

$$\vec{OB} = \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k},$$

$$\vec{OC} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}, \text{ and}$$

$$\vec{OD} = -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$$

Adjacent sides of the rectangle ABCD are \vec{AB} and \vec{BC} such that

$$\vec{AB} = (1 + 1)\hat{i} + \left(\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4 - 4)\hat{k} = 2\hat{j}$$

$$\vec{BC} = (1 - 1)\hat{i} + \left(-\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4 - 4)\hat{k} = -\hat{j}$$

Cross product between these two vectors will be,

$$\vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$$= \hat{k}(-2)$$

$$= -2\hat{k}$$

Since area of parallelogram having the adjacent sides \vec{a} and \vec{b} is $|\vec{a} \times \vec{b}|$, so area of the rectangle ABCD is $|\vec{AB} \times \vec{BC}| = 2$ square units.

Hence, the correct answer is option C.

Miscellaneous Exercise

1.

Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.

Ans - Unit vector is $\vec{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$ where θ is angle with positive X axis.

$$\vec{r} = \cos 30^\circ \hat{i} + \sin 30^\circ \hat{j} = \frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j}$$

2.

Find the scalar components and magnitude of the vector joining the points P(x_1, y_1, z_1) and Q : (x_2, y_2, z_2)

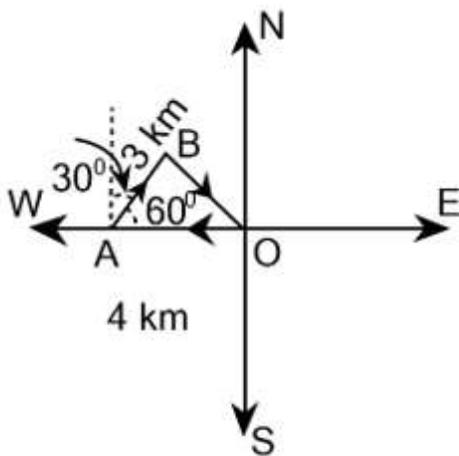
Ans - $\overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

3.

A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

Ans -



$$\overrightarrow{OA} = -4\hat{i}$$

$$\overrightarrow{OB} = \hat{i}|\overrightarrow{OB}|\cos 60^\circ + \hat{j}|\overrightarrow{OB}|\sin 60^\circ$$

$$= i3 \times \frac{1}{2} + j3 \times \frac{\sqrt{3}}{2}$$

$$= \frac{3}{2}i + \frac{3\sqrt{3}}{2}j$$

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

$$= (-4i) + \left(\frac{3}{2}i + \frac{3\sqrt{3}}{2}j\right)$$

$$= \left(-4 + \frac{3}{2}\right)i + \frac{3\sqrt{3}}{2}j$$

$$= \left(\frac{-8+3}{2}\right)i + \frac{3\sqrt{3}}{2}j$$

$$= \frac{-5}{2}i + \frac{3\sqrt{3}}{2}j$$

4.

If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

Ans - In $\triangle ABC$, $\overrightarrow{CB} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$

$\vec{a} = \vec{b} + \vec{c}$, by triangle law of addition for vectors.

$|\vec{a}| < |\vec{b}| + |\vec{c}|$, by triangle inequality law of lengths.

Hence, it's not true that $|\vec{a}| < |\vec{b}| + |\vec{c}|$

5.

Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ unit vector.

Ans - $|x(\hat{i} + \hat{j} + \hat{k})| = 1$

$$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1$$

$$\Rightarrow \sqrt{3x^2} = 1$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{3}}$$

6.

Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

$$\text{Ans - } \vec{c} = \vec{a} + \vec{b} = (2 + 1)\hat{i} + (3 - 2)\hat{j} + (-1 + 1)\hat{k} = 3\hat{i} + \hat{j}$$

$$|\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9 + 1} = \sqrt{10}$$

$$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{(3\hat{i} + \hat{j})}{\sqrt{10}}$$

So, a vector of magnitude 5 and parallel to the resultant of \vec{a} and \vec{b} is a $\pm 5(\hat{c}) = \pm 5\left(\frac{1}{\sqrt{10}}(3\hat{i} + \hat{j})\right) = \pm \frac{3\sqrt{10}}{2}\hat{i} \pm \frac{\sqrt{10}}{2}\hat{j}$

7.

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

Ans -

$$\begin{aligned} 2\vec{a} - \vec{b} + 3\vec{c} &= 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k}) \\ &= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k} \\ &= 3\hat{i} - 3\hat{j} + 2\hat{k} \end{aligned}$$

$$|2\vec{a} - \vec{b} + 3\vec{c}| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9 + 9 + 4} = \sqrt{22}$$

Thus, required unit vector is

$$\frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} = \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$$

8.

Show that the points A(1, -2, -8), B(5, 0, -2) and C(11, 3, 7) are collinear, and find the ratio in which B divides AC.

$$\text{Ans } \overrightarrow{AB} = (5 - 1)\hat{i} + (0 + 2)\hat{j} + (-2 + 8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\overrightarrow{BC} = (11 - 5)\hat{i} + (3 - 0)\hat{j} + (7 + 2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\overrightarrow{AC} = (11 - 1)\hat{i} + (3 + 2)\hat{j} + (7 + 8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k}$$

$$|\overrightarrow{AB}| = \sqrt{4^2 + 2^2 + 6^2} = \sqrt{16 + 4 + 36} = \sqrt{56} = 2\sqrt{14}$$

$$|\overrightarrow{BC}| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{36 + 9 + 81} = \sqrt{126} = 3\sqrt{14}$$

$$\begin{aligned} |\overrightarrow{AC}| &= \sqrt{10^2 + 5^2 + 15^2} = \sqrt{100 + 25 + 225} = \sqrt{350} \\ &= 5\sqrt{14} \end{aligned}$$

$$\therefore |\overrightarrow{AC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$$

So, the points are collinear.

$$\text{Let B divide AC in ratio } \lambda : 1. \overrightarrow{OB} = \frac{\lambda \overrightarrow{OC} + \overrightarrow{OA}}{(\lambda + 1)}$$

$$\Rightarrow 5\hat{i} - 2\hat{k} = \frac{\lambda(11\hat{i} + 3\hat{j} + 7\hat{k}) + (\hat{i} - \hat{j} - 8\hat{k})}{\lambda + 1}$$

$$\Rightarrow (\lambda + 1)5\hat{i} - 2\hat{k} = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - \hat{j} - 8\hat{k}$$

$$\begin{aligned} \Rightarrow 5(\lambda + 1)\hat{i} - 2(\lambda + 1)\hat{k} \\ = (11\lambda + 1)\hat{i} + (3\lambda - 2)\hat{j} + (7\lambda - 8)\hat{k} \end{aligned}$$

$$\Rightarrow \lambda = \frac{2}{3}$$

So, the required ration is 2 : 3

9.

Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1 : 2. Also, show that P is the mid point of the line segment RQ.

$$\text{Ans - } \vec{OP} = 2\vec{a} + \vec{b}, \vec{OQ} = \vec{a} - 3\vec{b}$$

$$\vec{OR} = \frac{2(2\vec{a} + \vec{b}) - (\vec{a} - 3\vec{b})}{2 - 1} = \frac{4\vec{a} + 2\vec{b} - \vec{a} - 3\vec{b}}{1} = 3\vec{a} + 5\vec{b}$$

SO, the position vector of R is $3\vec{a} + 5\vec{b}$

$$\text{Position vector of midpoint of } RQ = \frac{\vec{OQ} + \vec{OR}}{2}$$

$$= \frac{(a\sqrt{6}) + (3\vec{a} + 5\vec{b})}{2}$$

$$= 2\vec{a} + \vec{b}$$

$$= \vec{OP}$$

Thus, P is midpoint of line segment RQ.

10.

The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

Ans - Diagonal of a parallelogram is $\vec{a} + \vec{b}$

$$\vec{a} + \vec{b} = (2 + 1)\hat{i} + (-4 - 2)\hat{j} + (5 - 3)\hat{k} = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

$$\Rightarrow \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{3^2 + (-6)^2 + 2^2}}$$

$$= \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{9 + 36 + 4}}$$

$$= \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{7}$$

$$= \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 3 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(12 + 10) - \hat{j}(-6 - 5) + \hat{k}(-4 + 4)$$

$$= 22\hat{i} + 11\hat{j}$$

$$= 11(2\hat{i} + \hat{j})$$

$$\therefore |\vec{a} \times \vec{b}| = 11\sqrt{2^2 + 1^2} = 11\sqrt{5}$$

Hence area of the parallelogram is $11\sqrt{5}$ sq units.

11.

Show that the direction cosines of a vector equally inclined

to the axes OX, OY and OZ are $\pm \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Ans - Let a vector be equal inclined to OX, OY, and OZ at an angle α

Direction cosines of the vector are $\cos \alpha$, $\cos \alpha$ and $\cos \alpha$.

$$\cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$$

$$\Rightarrow 3 \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = \frac{1}{3}$$

$$\Rightarrow \cos \alpha = \pm \frac{1}{\sqrt{3}}$$

Hence direction cosines of vector are $\pm \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$.

12.

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} and $\vec{c} \cdot \vec{d} = 15$

Ans - $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$

$$\vec{d} \cdot \vec{a} = 0 \Rightarrow d_1 + 4d_2 + 2d_3 = 0$$

$$\vec{d} \cdot \vec{b} = 0 \Rightarrow 3d_1 - 2d_2 + 7d_3 = 0$$

$$\vec{c} \cdot \vec{d} = 15 \Rightarrow 2d_1 - d_2 + 4d_3 = 15$$

Solving these equations we get,

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3}, d_3 = -\frac{70}{3}$$

$$\therefore \vec{d} = \frac{160}{3}\hat{i} + \frac{5}{3}\hat{j} + \frac{70}{3}\hat{k} = \frac{1}{3}(160\hat{i} + 5\hat{j} + 70\hat{k})$$

13.

The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ .

$$\text{Ans} - (2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k}) = (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

So, unit vector along $(2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k})$

$$= \left(\frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} \right)$$

$$(\hat{i} + \hat{j} + \hat{k}) \cdot \left(\frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} \right) = 1$$

$$\Rightarrow \frac{(2 + \lambda) + 6 - 2}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \sqrt{\lambda^2 + 4\lambda + 44} = \lambda + 6$$

$$\Rightarrow \lambda^2 + 4\lambda + 44 = (\lambda + 6)^2$$

$$\Rightarrow \lambda^2 + 4\lambda + 44 = \lambda^2 + 12\lambda + 36$$

$$\Rightarrow 8\lambda - 8$$

$$\Rightarrow \lambda = 1$$

14.

If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to \vec{a}, \vec{b} and \vec{c} .

$$\text{Ans} - \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

$$|\vec{a}| = |\vec{b}| = |\vec{c}|$$

Let $\vec{a} + \vec{b} + \vec{c}$ be inclined to \vec{a}, \vec{b} and \vec{c} at angles θ_1, θ_2 and θ_3 respectively

$$\cos\theta_1 = \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{a}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|}$$

$$= \frac{\vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|} = \frac{|\vec{a}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|} = \frac{|\vec{a}|}{|\vec{a} + \vec{b} + \vec{c}|}$$

$$\cos\theta_2 = \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{b}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{b}|}$$

$$= \frac{\vec{a}\vec{b} + \vec{b}\vec{b} + \vec{c}\vec{b}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{b}|} = \frac{|\vec{b}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{b}|} = \frac{|\vec{b}|}{|\vec{a} + \vec{b} + \vec{c}|}$$

$$\cos\theta_3 = \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{c}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|}$$

$$= \frac{\vec{a}\vec{c} + \vec{b}\vec{c} + \vec{c}\vec{c}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} = \frac{|\vec{c}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} = \frac{|\vec{c}|}{|\vec{a} + \vec{b} + \vec{c}|}$$

Since, $|\vec{a}| = |\vec{b}| = |\vec{c}|$, $\cos\theta_1 = \cos\theta_2 = \cos\theta_3$

$$\Rightarrow \theta_1 = \theta_2 = \theta_3$$

15.

Prove that, $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$ if and only if \vec{a}, \vec{b} are perpendicular, given $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$.

$$\text{Ans - } (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2$$

$$\Rightarrow |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$$

$$\Rightarrow 2\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0$$

So \vec{a} and \vec{b} are perpendicular.

16.

If θ is the angle between two vectors \vec{a} and \vec{b} , then $\vec{a} \cdot \vec{b} \geq 0$ only when

(A) $0 < \theta < \frac{\pi}{2}$

(B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$

(D) $0 \leq \theta \leq \pi$

$$\text{Ans} - \vec{a} \cdot \vec{b} \geq 0$$

$$\Rightarrow |\vec{a}| |\vec{b}| \cos \theta \geq 0$$

$$\Rightarrow \cos \theta \geq 0 \quad \because [|\vec{a}| \geq 0 \text{ and } |\vec{b}| \geq 0]$$

$$\Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$$

$$\vec{a} \cdot \vec{b} \geq 0 \text{ if } 0 \leq \theta \leq \frac{\pi}{2}$$

Hence correct answer is option B.

17.

Let \vec{a} and \vec{b} be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if

$$(A) \theta = \frac{\pi}{4} \quad (B) \theta = \frac{\pi}{3} \quad (C) \theta = \frac{\pi}{2} \quad (D) \theta = \frac{2\pi}{3}$$

$$\text{Ans} - |\vec{a}| = |\vec{b}| = 1$$

$$|\vec{a} + \vec{b}| = 1$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = 1$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} = 1$$

$$\Rightarrow |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = 1$$

$$\Rightarrow 1^2 + 2|\vec{a}| |\vec{b}| \cos \theta + 1^2 = 1$$

$$\Rightarrow 1 + 2 \cdot 1 \cdot 1 \cos \theta + 1 = 1$$

$$\Rightarrow \cos \theta = -\frac{1}{2}$$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

$$\text{So, } \vec{a} + \vec{b} \text{ is unit vector if } \theta = \frac{2\pi}{3}$$

Hence correct answer is option D.

18.

The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is

- (A) 0 (B) -1 (C) 1 (D) 3

Ans - $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$

$$= \hat{i} \cdot \hat{i} + \hat{j} \cdot (-\hat{j}) + \hat{k} \cdot \hat{k}$$

$$= 1 - 1 + 1$$

$$= 1$$

Hence correct answer is option C.

19.

If θ is the angle between any two vectors \vec{a} and \vec{b} , then

$|\vec{a}\vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to

- (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

Ans - $|\vec{a}\vec{b}| = |\vec{a} \times \vec{b}|$

$$\Rightarrow |\vec{a}||\vec{b}| \cos \theta = |\vec{a}||\vec{b}| \sin \theta$$

$$\Rightarrow \cos \theta = \sin \theta$$

$$\Rightarrow \tan \theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence correct answer is option B.