NCERT Solutions for Class 12 Maths Chapter 1 – Relations and Functions

Exercise 1.1

1.

Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set $A = \{1, 2, 3, ..., 13, 14\}$ defined as
$R = \{(x, y) : 3x - y = 0\}$

(ii) Relation R in the set N of natural numbers defined as
$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as
$R = \{(x, y) : y \text{ is divisible by } x\}$

(iv) Relation R in the set Z of all integers defined as
$R = \{(x, y) : x - y \text{ is an integer}\}$

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x, y) : x \text{ and } y \text{ work at the same place}\}$
(b) $R = \{(x, y) : x \text{ and } y \text{ live in the same locality}\}$
(c) $R = \{(x, y) : x \text{ is exactly 7 cm taller than } y\}$
(d) $R = \{(x, y) : x \text{ is wife of } y\}$
(e) $R = \{(x, y) : x \text{ is father of } y\}$

Ans - **(i)** Given relation R = {(1, 3), (2, 6), (3, 9), (4, 12)}

(1, 1), (2, 2) ... and (14, 14) $\notin$ R

$\Rightarrow$ We conclude that R is not reflexive.

(1, 3) $\in$ R, but (3, 1) $\notin$ R [$\because 3(3) - 1 \neq 0$]

$\Rightarrow$ We conclude that R is not symmetric.

(1, 3) and (3, 9) $\in$ R, but(1, 9) $\notin$ R. [$\because 3(1) - 9 \neq 0$]

$\Rightarrow$ We conclude that R is not transitive.

Hence, relation R is not reflexive, symmetric or transitive.

(ii) Given relation R = {(1, 6), (2, 7), (3, 8)}

(1, 1) $\notin$ R

$\Rightarrow$ We conclude that R is not reflexive.

(1, 6) $\in$ R but (6, 1) $\notin$ R.

$\Rightarrow$ We conclude that R is not symmetric.

In the given relation R there is not any ordered pair such that (x, y) and (y, z) both $\in$R, therefore we can say that (x, z) cannot belong to R.

$\Rightarrow$ We conclude that R is not transitive.

Hence, relation R is not reflexive, symmetric or transitive.

(iii) Given relation R = { (x, y) : y is divisible by x }

We know that any number except 0 is divisible by itself, hence (x, x) $\in$ R.

$\Rightarrow$ We conclude that R is reflexive.

(2, 4) $\in$ R [$\because$ 4 is divisible by 2], but (4, 2) $\notin$ R [$\because$ 2 is not divisible by 4]

$\Rightarrow$ We conclude that R is not symmetric.

Assuming that (x, y) and (y, z) $\in$ R, y is divisible by x and z is divisible by y. Hence z is divisible by x i.e. (x, z)$\in$R.

$\Rightarrow$ We conclude that R is transitive.

Hence, relation R is reflexive and transitive but it is not symmetric.

(iv) Given relation R = { (x, y): x – y is an integer }

If x is an integer then (x, x) ∈ R because x – x = 0 is an integer.

⇒ We conclude that R is reflexive.

For x, y ∈ Z, if (x, y) ∈ R, then x – y is an integer and therefore (y – x) is also an integer i.e. (y, x) ∈ R

⇒ We conclude that R is symmetric.

For x, y, z ∈ Z, if (x, y) and (y, z) ∈ R, we can say that (x – y) and (y – z) are integers i.e. (x, z) ∈ R

⇒ We conclude that R is transitive.

Hence, relation R is reflexive, symmetric, and transitive.

(v) (a) Given relation R = { (x, y): x and y work at the same place }

Here (x, x) ∈ R

⇒ We conclude that R is reflexive.

If (x, y)∈R, then x and y work at same place. That means y and x also work at same place, i.e. (y, x)∈R.

⇒ We conclude that R is symmetric.

Assume (x, y), (y, z)∈R.

Then, we can say that x and y work at same place and y and z work at same place. Thus x and z also work at same place, i.e. (x, z) ∈ R.

⇒ We conclude that R is transitive.

Hence, relation R is reflexive, symmetric and transitive.

(v) (b) Given relation R = { (x, y): x and y live in the same locality }

Here (x, x) ∈ R

⇒ We conclude that R is reflexive.

If (x, y)∈R, then x and y live in the same locality. Which means y and x also live in the same locality, i.e. (y, x) ∈ R

⇒ We conclude that R is symmetric.

Assume (x, y) ∈ R and (y, z) ∈ R. Then, x and y live in the same locality and y and z also live in the same locality. Which means that x and z also live in the same locality, i.e. (x, z) ∈ R.

⇒ We conclude that R is transitive.

Hence, relation R is reflexive, symmetric and transitive.

(v) (c) Given relation R = { (x, y): x is exactly 7 cm taller than y }

Here (x, x) ∉ R.

⇒ We conclude that R is not reflexive.

Let (x, y)∈R , Since x is exactly 7 cm taller than y, therefore y is obviously not taller than x, so, (y, x)∉R.

⇒ We conclude that R is not symmetric.

Assuming that (x, y), (y, z)∈R, we can say that x is exactly 7 cm taller than yand y is exactly 7 cm taller than z. Which means that x is exactly 14 cm taller than z. So, (x, z)∉R.

⇒ We conclude that R is not transitive.

Hence, relation R is not reflexive, symmetric or transitive.

(v) (d) Given relation R = { (x, y): x is the wife of y } .

Here (x, x) ∉ R

⇒ We conclude that R is not reflexive.

Let (x, y)∈R , Since x is the wife of y, then y can't be wife of x, i.e. (y, x) ∉ R.

⇒ We conclude that R is not symmetric.

Assuming that (x, y), (y, z)∈R, we can say that x is the wife of y and y is the wife of z, which is not possible i.e. (x, z) ∉ R

⇒ We conclude that R is not transitive.

Hence, relation R is not reflexive, symmetric or transitive.

(v) (e) Given relation R = { (x, y): x is the father of y }

Here (x, x) ∉ R

⇒ We conclude that R is not reflexive.

Let (x, y)∈R , Since x is the father of y, then y can't be the father of x, i.e. (y, x) ∉ R.

⇒ We conclude that R is not symmetric.

Assuming that (x, y), (y, z)∈R, we can say that x is the father of yand y is the father of z, then x is not the father of z, i.e. (x, z) ∉ R.

⇒ We conclude that R is not transitive.

Hence, relation R is not reflexive, symmetric or transitive.

2.

Show that the relation R in the set R of real numbers, defined as R = {(a, b) : a ≤ b²} is neither reflexive nor symmetric nor transitive.

Ans - Given relation $R = \{(a, b): a \leq b^2\}$

Since $(\frac{1}{2}, \frac{1}{2}) \notin R$. [$\because \frac{1}{2}$ is not less than $\frac{1}{4}$]

$\Rightarrow$ We conclude that R is not reflexive.

$(1, 4) \in R$ as $1 \leq 4^2$, but $(4, 1) \notin R$ as 4 is not less than 1^2.

$\Rightarrow$ We conclude that R is not symmetric.

Assuming that $(3, 2), (2, 1.5) \in R$,
$\Rightarrow 3 \leq 2^2 = 4$ and $2 \leq (1.5)^2 = 2.25$ but 3 is not less than $(1.5)^2 = 2.25$.

$\Rightarrow$ We conclude that R is not transitive.

Hence, relation R is not reflexive, symmetric or transitive.

3.

Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

Ans - Given relation $R = \{(a, b): b=a+1\}$ defined in the set $A=\{1, 2, 3, 4, 5, 6\}$.

$\Rightarrow R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

Here $(a, a) \notin R, a \in A$.

[$\because (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$]

$\Rightarrow$ We conclude that R is not reflexive.

Here $(1, 2) \in R$, but $(2, 1) \notin R$.

$\Rightarrow$ We conclude that R is not symmetric.

Here $(1, 2), (2, 3) \in R$, but $(1, 3) \notin R$.

$\Rightarrow$ We conclude that R is not transitive.

Hence, relation R is not reflexive, symmetric or transitive.

4.

Show that the relation R in R defined as $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.

Ans - Given relation $R = \{(a, b): a \leq b\}$.

Here $(a, a) \in R$.

$\Rightarrow$ We conclude that R is reflexive.

$(2, 4) \in R$ [$\because 2 \leq 4$], but $(4, 2) \notin R$ [$\because$ 4 is not ≤ 2]

$\Rightarrow$ We conclude that R is not symmetric.

Assuming that $(a, b), (b, c) \in R$, $a \leq b$ and $b \leq c$, then $a \leq c$.

$\Rightarrow$ We conclude that R is transitive.

Hence, relation R is reflexive and transitive but not symmetric.

5.

Check whether the relation R in R defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

Ans - Given relation $R = \{(a, b): a \leq b^3\}$

Here $(\frac{1}{2}, \frac{1}{8}) \notin R$. [$\because \frac{1}{2}$ is not $\leq \frac{1}{8}$]

$\Rightarrow$ We conclude that R is not reflexive.

$(1, 4) \in R$ as $1 \leq 4^3$, but $(4, 1) \notin R$ as 4 is not $\leq 1^3$.

$\Rightarrow$ We conclude that R is not symmetric.

Assuming that $(3, \frac{3}{2}), (\frac{3}{2}, \frac{6}{5}) \in R$, so, $3 \leq \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} \leq \left(\frac{6}{5}\right)^3$ but 3 is not $\leq \left(\frac{6}{5}\right)^3 \notin R$.

$\Rightarrow$ We conclude that R is not transitive.

Hence, relation R is not reflexive, symmetric or transitive.

6.

Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

Ans - Given relation R = {(1, 2), (2, 1)} on the set A={1, 2, 3}.

Since (1, 1), (2, 2), (3, 3)∉R

⇒ We conclude that R is not reflexive.

Since, (1, 2) ∈ R and (2, 1) ∈ R.

⇒ We conclude that R is symmetric.

Since, (1, 2) ∈ R and (2, 1) ∈ R, but (1, 1) ∉ R.

⇒ We conclude that R is not transitive.

Hence, relation R is symmetric but neither reflexive nor transitive.

7.

Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y) : x and y have same number of pages} is an equivalence relation.

Ans - Given relation R = { (x,y): x and y have the same number of pages }

(x, x) ∈ R, because x and x have same number of pages.

⇒ We conclude that R is reflexive.

Let (x, y) ∈ R, so x and y have the same number of pages, then (y, x) ∈ R because y and x will also have same number of pages.

⇒ We conclude that R is symmetric.

Let (x, y) ∈ R and (y, z) ∈ R. So x and y have same number of pages and y and z also have same number of pages. Therefore, x and z will also have the same number of pages. i.e, (x, z) ∈ R.

⇒ We conclude that R is transitive.

Hence, relation R is an equivalence relation.

8.

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

Ans - Let a ∈ A,

So, |a – a| = 0 (which is an even number).

⇒ We conclude that R is reflexive.

Let (a, b) ∈ R,

|a – b| is even, hence |a – b| and |b – a| are both even

⇒ (b, a) ∈ R

⇒ We conclude that R is symmetric.

Let (a, b) ∈ R and (b, c) ∈ R,

⇒ |a – b| is even and |b – c| is even

⇒ |a – c| is even.

⇒ (a, c) ∈ R

⇒ We conclude that R is transitive.

Hence, relation R is an equivalence relation.

All elements in the set {1, 3, 5} are odd. As a result, the modulus of the difference between any two elements will be even. So, all of the items in this set are related to one another.

All elements of {2, 4} are even, whereas all elements of {1, 3, 5} are odd, hence no element of {1, 3, 5} is connected to any element of {2, 4}.

Hence, the absolute value of the difference between the two components from each of these subsets will not yield an even value.

9.

Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by
(i) $R = \{(a, b) : |a - b|$ is a multiple of 4$\}$
(ii) $R = \{(a, b) : a = b\}$
is an equivalence relation. Find the set of all elements related to 1 in each case.

Ans - (i) Given set $A = \{x \in Z: 0 \leq x \leq 12\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Given relation $R = \{(a, b) : |a - b|$ is a multiple of 4 $\}$.

Let $a \in A$,

$(a, a) \in R$ as $|a - a| = 0$ is a multiple of 4.

$\Rightarrow$ We conclude that R is reflexive.

Let $(a, b) \in R$

$\Rightarrow |a - b|$ is a multiple of 4.

$|b - a| = |-(a - b)|$ is a multiple of 4.

$\Rightarrow (b, a) \in R$

$\Rightarrow$ We conclude that R is symmetric.

$(a, b), (b, c) \in R$.

$\Rightarrow |a - b|$ is a multiple of 4 and $|b - c|$ is a multiple of 4.

$\Rightarrow (a - b)$ is a multiple of 4 and $(b - c)$ is a multiple of 4.

$(a - c) = (a - b) + (b - c)$ is a multiple of 4.

$\Rightarrow |a - c|$ is a multiple of 4.

$\Rightarrow (a, c) \in R$

$\Rightarrow$ We conclude that R is transitive.

Hence, relation R is an equivalence relation.

Set of elements related to 1 is $\{1, 5, 9\}$

(ii) Given relation R = {(a, b) : a = b}.

Let a ∈ A, then (a, a) ∈ R, since a = a.

⇒ We conclude that R is reflexive.

(a, b) ∈ R since a = b.

⇒ b = a, then (b, a) ∈ R

⇒ We conclude that R is symmetric.

Let (a, b), (b, c) ∈ R. Then a = b and b = c.

⇒ a = c, i.e. (a, c) ∈ R

⇒ We conclude that R is transitive.

Hence, relation R is an equivalence relation.

Set of elements related to 1 is {1}.

10.

Give an example of a relation. Which is
(i) Symmetric but neither reflexive nor transitive.
(ii) Transitive but neither reflexive nor symmetric.
(iii) Reflexive and symmetric but not transitive.
(iv) Reflexive and transitive but not symmetric.
(v) Symmetric and transitive but not reflexive.

Ans - **(i)** Let us assume relation R= {(5, 6), (6, 5)} in set A= {5, 6, 7}.

(5, 5), (6, 6), (7, 7) ∉ R.

⇒ We conclude that R is not reflexive.

(5, 6) ∈ R and (6, 5) ∈ R.

⇒ We conclude that R is symmetric.

(5, 6) ∈ R and (6, 5) ∈ R, but (5, 5) ∉ R.

⇒ We conclude that R is not transitive.

Hence, relation R is symmetric but not reflexive or transitive.

(ii) Let us assume the relation R = {(a, b) : a b}

Let a ∈ R, then (a, a) ∉ R [∵ a cannot be less than itself]

⇒ We conclude that R is not reflexive.

Let (1, 2) ∈ R [∵ $1 < 2$]

Since 2 is not less than 1, (2, 1)∉R.

⇒ We conclude that R is not symmetric.

Let (a, b), (b, c) ∈ R.

Here $a < b$ and $b < c$. So we ge $a < c$

⇒ (a, c) ∈ R

⇒ We conclude that R is transitive.

Hence, relation R is transitive but not reflexive and symmetric.

(iii) Let us assume relation R = {(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)} in set A= {4, 6, 8}.

(4, 4), (6, 6), (8, 8) ∈ R

⇒ We conclude that R is reflexive.

(4, 6), (6, 4), (6, 8), (8, 6) ∈ R

⇒ We conclude that R is symmetric.

(4, 6), (6, 8) ∈ R, but (4, 8) ∉ R.

⇒ We conclude that R is not transitive.

Hence, relation R is reflexive and symmetric but not transitive.

(iv) Let us assume relation R = {(a, b) : $a^3 \geq b^3$}.

Since (a, a) ∈ R.

⇒ We conclude that R is reflexive.

Since (2, 1) ∈ R, but (1, 2) ∉ R,

⇒ We conclude that R is not symmetric.

Let (a, b), (b, c) ∈ R

⇒ $a^3 \geq b^3$ and $b^3 \geq c^3$. So we get $a^3 \geq c^3$

⇒(a, c) ∈ R

⇒ We conclude that R is transitive.

Hence, relation R is reflexive and transitive but not symmetric.

(v) Let us assume relation R = {(–5, –6), (–6, –5), (–5, –5)} in set A = {–5, –6}.

(–6, –6) ∉ R

⇒ We conclude that R is not reflexive.

Since (–5, –6) ∈ R and (–6, –5) ∈ R.

⇒ We conclude that R is symmetric.

Since (–5, –6), (–6, –5) ∈ R and (–5, –5) ∈ R.

⇒ We conclude that R is transitive.

Hence, relation R is symmetric and transitive but not reflexive.

11.

Show that the relation R in the set A of points in a plane given by R = {(P, Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.

Ans - Given relation R = {(P, Q) : Distance of P from the origin is the same as the distance of Q from the origin}

(P, P) ∈ R.

⇒ We conclude that R is reflexive.

Let (P, Q) ∈ R; the distance of P from the origin is equivalent to the distance of Q from the origin, and similarly, the distance of Q from the origin is equal to the distance of P from the origin. i.e, (Q, P) ∈ R.

⇒ We conclude that R is symmetric.

Let (P, Q), (Q, S) ∈ R.

P's distance from the origin is equal to Q's distance from the origin, and Q's distance from the origin equals S's distance from the origin. So, distance of S from the origin will equal the distance of P from the origin. i.e, (P, S)∈R.

⇒ We conclude that R is transitive.

Hence, relation R is an equivalence relation.

The set of points connected to P ≠ (0, 0) are those whose distance from the origin is equal to P's distance from the origin. These points form a circle with the centre as the origin, passing through P.

12.

Show that the relation R defined in the set A of all triangles as R = {(T_1 , T_2) : T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Ans - Given relation $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$ since every triangle is similar to itself.

⇒ We conclude that R is reflexive.

If $(T_1, T_2) \in R$, then T_1 is similar to T_2.

⇒ T_2 is similar to T_1.

⇒ $(T_2, T_1) \in R$

⇒ We conclude that R is symmetric.

Let $(T_1, T_2), (T_2, T_3) \in R$.

⇒ T_1 is similar to T_2 and T_2 is similar to T_3.

⇒ T_1 is similar to T_3.

⇒ $(T_1, T_3) \in R$

⇒ We conclude that R is transitive.

Hence, relation R is an equivalence relation.

$$\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$$

Since the corresponding sides of triangles T_1 and T_3 are in the same ratio, triangle T_1 is similar to triangle T_3.

Hence, T_1 is related to T_3.

13.

Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same number of sides}\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Ans - Given relation R = { (P1, P2): P1 and P2 have same number of sides } .

Since $(P_1, P_1) \in R$, as same polygon has same number of sides.

⇒ We conclude that R is reflexive.

Let $(P_1, P_2) \in R$. Then P_1 and P_2 have same number of sides.

⇒ P_2 and P_1 have same number of sides.

⇒ $(P_2, P_1) \in R$

⇒ We conclude that R is symmetric.

Let $(P_1, P_2), (P_2, P_3) \in R$. Then P_1 and P_2 have same number of sides and P_2 and P_3 have same number of sides.

⇒ P_1 and P_3 have same number of sides.

⇒ $(P_1, P_3) \in R$

⇒ We conclude that R is transitive.

Hence, relation R is an equivalence relation.

Elements in A related to right-angled triangle (T) with sides 3, 4 and 5 are the polygons having 3 sides.

14.

Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1 , L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Ans - Given relation $R = \{(L_1, L_2): L_1 \text{ is parallel to } L_2\}$.

Any line L_1 is parallel to itself, so, $(L_1, L_1) \in R$.

⇒ We conclude that R is reflexive.

Let $(L_1, L_2) \in R$. Then L_1 is parallel to L_2, therefore L_2 is parallel to L_1.

⇒ $(L_2, L_1) \in R$

⇒ We conclude that R is symmetric.

Let $(L_1, L_2), (L_2, L_3) \in R$. L_1 is parallel to L_2 and L_2 is parallel to L_3, therefore L_1 is parallel to L_3

⇒ $(L_1, L_3) \in R$

⇒ We conclude that R is transitive.

Hence, relation R is an equivalence relation.

Set of all lines related to the line $y = 2x + 4$ is the set of all lines that are parallel to the line $y = 2x + 4$.

Slope of line $y = 2x + 4$ is $m = 2$. Therefore, lines parallel to the given line are of the form $y = 2x + c$, where $c \in R$.

15.

Let R be the relation in the set {1, 2, 3, 4} given by
R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}.
Choose the correct answer.
(A) R is reflexive and symmetric but not transitive.
(B) R is reflexive and transitive but not symmetric.
(C) R is symmetric and transitive but not reflexive.
(D) R is an equivalence relation.

Ans - Given relation $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$.

Since $(a, a) \in R$, for every $a \in \{1, 2, 3, 4\}$

$\Rightarrow$ We conclude that R is reflexive.

Since $(1, 2) \in R$, but $(2, 1) \notin R$.

$\Rightarrow$ We conclude that R is not symmetric.

$(a, b), (b, c) \in R$ then $(a, c) \in R$ for all $a, b, c \in \{1, 2, 3, 4\}$.

$\Rightarrow$ We conclude that R is transitive.

Hence, relation R is reflexive and transitive but not symmetric.

Correct answer is option (B).

16.

Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Choose the correct answer.

(A) $(2, 4) \in R$ (B) $(3, 8) \in R$

(C) $(6, 8) \in R$ (D) $(8, 7) \in R$

Ans - Given relation $R = \{(a, b): a = b - 2, b > 6\}$

Considering option (A).

Since, $b > 6$, so, $(2, 4) \notin R$.

Considering option (B).

Since $3 \neq 8 - 2$, so $(3, 8) \notin R$.

Considering option (C).

Since $8 > 6$ and $6 = 8 - 2$, so $(6, 8) \in R$.

Hence, correct answer is option (C).

Exercise 1.2

1.

Show that the function $f: R_* \rightarrow R_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R∗ is the set of all non-zero real numbers. Is the result true, if the domain R_* is replaced by N with co-domain being same as R_*?

Ans - Given that $f: R_* \rightarrow R_*$ is defined by $f(x) = \frac{1}{x}$.

One – One:

$f(x) = f(y)$

$\Rightarrow \frac{1}{x} = \frac{1}{y}$

$\Rightarrow x = y$

$\therefore$ f is one-one.

Onto:

It is clear that for $y \in R_*$ there exists $x = \frac{1}{y} \in R_*$ (Exists as $y \neq R_*$) Exists as $y \neq 0$ such that $f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y$

Now, consider function $g: N \rightarrow R_*$ defined by

$g(x) = \frac{1}{x}$.

We have,

$g(x_1) = g(x_2)$

$\Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_1 = x_2$

$\therefore$ g is one-one.

It is clear that g is not onto as for $1.2 \in R_*$ there does not exist any x in N such that

$g(x) = \frac{1}{1.2}$

Hence, the function g is one-one but not onto.

2.

Check the injectivity and surjectivity of the following functions:
(i) f : N → N given by $f(x) = x^2$
(ii) f : Z → Z given by $f(x) = x^2$
(iii) f : R → R given by $f(x) = x^2$
(iv) f : N → N given by $f(x) = x^3$
(v) f : Z → Z given by $f(x) = x^3$

Ans - **(i)** Here, f : N → N is given by $f(x) = x^2$

For x, y ∈ N, f(x) = f(y)

$\Rightarrow x^2 = y^2$

$\Rightarrow x = y$

⇒ f is injective.

Now, 2∈N. But, there does not exist any x in N such that $f(x) = x^2 = 2$

⇒ f is not surjective.

Hence, function f is injective but not surjective.

(ii) Here, f : Z → Z is given by $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

⇒ f is not injective.

Now, $-2 \in Z$. But, there does not exist any element x∈Z such that $f(x) = -2$ or $x^2 = -2$

⇒ f is not surjective.

Hence, function f is neither injective nor surjective.

(iii) Here, f : R → R is given by $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

⇒ f is not injective.

Now, $-2 \in R$. But, there does not exist any element x∈R such that $f(x) = -2$ or $x^2 = -2$.

⇒ f is not surjective.

Hence, function f is neither injective nor surjective.

(iv) Here, f : N → N given by $f(x) = x^3$

For x, y ∈ N, f(x) = f(y)

$\Rightarrow x^3 = y^3$

$\Rightarrow x = y$

⇒ f is injective.

Now, 2 ∈ N. But, there does not exist any element x ∈ N such that f(x)=2 or x3=2

⇒ f is not surjective

Hence, function f is injective but not surjective.

(v) Here, f : Z → Z is given by $f(x) = x^3$

For x, y ∈ Z, f(x) = f(y)

$\Rightarrow x^3 = y3$

$\Rightarrow x = y$

⇒ f is injective.

Now, 2 ∈ Z. But, there does not exist any element x ∈ Z such that f(x) = 2 or $x^3 = 2$

⇒ f is not surjective.

Hence, function f is injective hut not surjective.

3.

Prove that the Greatest Integer Function f : R → R, given by f(x) = [x], is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

Ans - Here, $f : R \rightarrow R$ is given by, $f(x) = [x]$

It is known that,

$f(1.2) = [1.2] = 1$

$f(1.9) = [1.9] = 1$

$\Rightarrow f(1.2) = f(1.9)$, but $1.2 \neq 1.9$

$\Rightarrow$ f is not one-one.

Consider $0.7 \in R$

Known that $f(x) = [x]$ is always an integer.

Thus, there does not exist any element $x \in R$ such that $f(x) = 0.7$

$\Rightarrow$ f is not onto.

Hence, greatest integer function is neither one-one nor onto.

4.

Show that the Modulus Function $f : R \rightarrow R$, given by $f(x) = | x |$, is neither oneone nor onto, where $| x |$ is x, if x is positive or 0 and $| x |$ is $-x$, if x is negative

Ans - Here, $f : R \rightarrow R$ is given by

$$f(x) = |x| = \begin{cases} x, if\ x \geq 0 \\ -x, if\ x < 0 \end{cases}$$

It is clear that,

$f(-1) = |-1| = 1$

$f(1) = |1| = 1$

$\Rightarrow f(-1) = f(1)$, but $-1 \neq 1$

$\Rightarrow$ f is not one-one.

Let $-1 \in R$.

It is known that $f(x) = |x|$ is always non-negative.

Thus, there does not exist any element x in domain R such that $f(x) = |x| = -1$

$\Rightarrow$ f is not onto.

Hence, modulus function is neither one-one nor onto.

5.

Show that the Signum Function $f : R \rightarrow R$, given by

$$\boldsymbol{f(x) = \begin{cases} 1, if\ x > 0 \\ 0, if\ x = 0 \\ -1, if\ x < 0 \end{cases}}$$

is neither one-one nor onto.

Ans - Here, f : R → R, given by

$$f(x) = \begin{cases} 1, if\ x > 0 \\ 0, if\ x = 0 \\ -1, if\ x < 0 \end{cases}$$

It is known that, f(1) = f(2) = 1 but 1≠2.

⇒ f is not one-one.

As f(x) takes only three values (1, 0, or −1), for the element −2 in the co-domain R, there does not exist any x in domain R such that f(x) = −2.

⇒ f is not onto

Hence, Signum function is neither one-one nor onto.

6.

Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

Ans - Given that,

A = {1,2,3}

B = {4,5,6,7}

f : A → B is defined as f = {(1,4), (2,5), (3,6)}

⇒ f(1) = 4
⇒ f(2) = 5
⇒ f(3) = 6

It is seen that the images of distinct elements of A under f are distinct.

Hence, function f is one-one.

7.

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.
(i) f : R → R defined by f(x) = 3 – 4x
(ii) f : R → R defined by f(x) = 1 + x^2

Ans - **(i)** Here, $f : R \rightarrow R$ is defined as $f(x) = 3 - 4x$.

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow 3 - 4x_1 = 3 - 4x_2$

$\Rightarrow -4x_1 = -4x_2$

$\Rightarrow x_1 = x_2$

$\Rightarrow$ f is one-one.

For any real number (y) in R, there exists $\frac{3-y}{4}$ in R such that

$$f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = y$$

$\Rightarrow$ f is onto.

Hence, function f is bijective.

(ii) Here, $f : R \rightarrow R$ defined by $f(x) = 1+x_2$

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow 1+ (x_1)^2 = 1+(x_2)^2$

$\Rightarrow (x_1)^2 = (x_2)^2$

$\Rightarrow x_1 = x_2$

Thus, $f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$.

For example, $f(1) = f(-1) = 2$

$\Rightarrow$ f is not one-one.

Consider an element -2 in co-domain R.

It is seen that $f(x) = 1 + x^2$ is positive for all $x \in R$.

Thus, there does not exist any x in domain R such that
$f(x) = -2$.

$\Rightarrow$ f is not onto.

Hence, function f is neither one-one nor onto.

8.

Ans - $f : A \times B \to B \times A$ is defined as $f(a, b) = (b, a)$

Let $(a_1, b_1), (a_2, b_2) \in A \times B$ such that $f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

$\Rightarrow$ f is one-one.

Let $(b, a) \in B\ A$ be any element. Then, there exists
$(a, b) \in A \times B$ such that $f(a, b) = (b, a)$.[By definition of f]

$\Rightarrow$ f is onto.

Hence, function f is bijective.

9.

Let f : N → N be defined by

$$\mathbf{f(n) = \begin{cases} \dfrac{n+1}{2}, \text{if n is odd} \\ \dfrac{n}{2}, \text{if n is even} \end{cases} \quad \text{for all } n \in N}$$

State whether the function f is bijective. Justify your answer.

Ans - By definition of f it can be observed that,

$$f(1) = \frac{1+1}{2} = 1$$

$$f(2) = \frac{2}{2} = 1$$

$\therefore f(1) = f(2)$, where $1 \neq 2$

$\Rightarrow f$ is not one-one.

Consider a natural number (n) in co-domain N.

Case I: n is odd

$\therefore n = 2r + 1$ for some $r \in N$. Then, there exists $4r + 1 \in N$ such that

$$f(4r + 1) = \frac{4r + 1 + 1}{2} = 2r + 1$$

Case II: n is even

$\therefore n = 2r$ for some $r \in N$. Then, there exists $4r \in N$ such that

$$f(4r) = \frac{4r}{2} = 2r$$

$\Rightarrow$ f is onto.

Hence, function f is not a bijective function.

10.

Let A = R – {3} and B = R – {1}. Consider the function f : A → B defined by

$$f(x) = \left(\frac{x-2}{x-3}\right)$$

Is f one-one and onto? Justify your answer.

Let $x, y \in A$ such that $f(x) = f(y)$

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

By cross multiplication,

$\Rightarrow (x - 2)(y - 3) = (y - 2)(x - 3)$

$\Rightarrow xy - 3x - 2y + 6 = xy - 2x - 3y + 6$

$\Rightarrow -3x - 2y = -2x - 3y$

$\Rightarrow x = y$

$\Rightarrow$ f is one-one.

Let $y \in B = R - \{1\}$. Then, $y \neq 1$.

Function f is onto if there exists $x \in A$ such that $f(x) = y$.

Function f is onto if there exists x ∈ A such that f(x) = y.

$f(x) = y$

$\Rightarrow \dfrac{x-2}{x-3} = y$

$\Rightarrow x - 2 = xy - 3y$

$\Rightarrow x(1 - y) = -3y + 2$

$\Rightarrow x = \dfrac{2-3y}{1-y} \in A \; [y \neq 1]$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$$

⇒ f is onto.

Hence, function f is one-one and onto.

11.

Let f : R → R be defined as f(x) = x^4. Choose the correct answer.
(A) f is one-one onto
(B) f is many-one onto
(C) f is one-one but not onto
(D) f is neither one-one nor onto.

Ans - Here, f : R → R is defined as $f(x) = x^4$.

Let x, y ∈ R such that f(x) = f(y).

$\Rightarrow x^4 = y^4$

$\Rightarrow x = \pm y$

f(x) = f(y) does not imply that x = y

For example, f(1) = f(−1) = 1

⇒ f is not one-one.

Consider an element 2 in co-domain R. It is clear that there does not exist any x in domain R such that f(x) = 2

⇒ f is not onto.

Hence, function f is neither one-one nor onto.

∴ Correct answer is option (D).

12.

Let f : R → R be defined as f(x) = 3x. Choose the correct answer.
(A) f is one-one onto
(B) f is many-one onto
(C) f is one-one but not onto
(D) f is neither one-one nor onto.

Ans - Here, f : R → R is defined as f(x) = 3x.

Let x, y ∈ R such that f(x) = f(y).

⇒ 3x = 3y

⇒ x = y

⇒ f is one-one.

Now, for any real number (y) in co-domain R, there exists $\frac{y}{3}$ in R such that

$$f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$$

⇒ f is onto.

Hence, function f is one-one and onto.

∴ Correct answer is option (A).

Miscellaneous Exercise

1.

Show that function $f: R \rightarrow \{x \in R: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one-one and onto function.

Ans - Let x, y∈R. For the function f to be one – one then,
f(x) = f(y)

$$\Rightarrow \frac{x}{1+|x|} = \frac{y}{1+|y|}$$

Let x be positive and y be negative

$$\Rightarrow \frac{x}{1+x} = \frac{y}{1+y}$$

$$\Rightarrow 2xy = x - y$$

Since x > y, we get x – y > 0.

But 2xy is negative.

∴ 2xy ≠ x – y.

As a result, it is impossible for x to be positive when y is negative. Similarly, negative x and positive y can be ruled out. So, x and y must be either positive or negative.

Let x and y be positive

f(x) = f(y)

$$\Rightarrow \frac{x}{1+x} = \frac{y}{1+y}$$

$$\Rightarrow x + xy = y + xy$$

$$\Rightarrow x = y$$

Similarly let x and y be negative

f(x) = f(y)

$$\Rightarrow \frac{x}{1+x} = \frac{y}{1+y}$$

$$\Rightarrow x + xy = y + xy$$

$$\Rightarrow x = y$$

∴ Function f is one – one.

For onto:

$y \in R$ such that $-1 < y < 1$.

Let y be negative then $x = \frac{y}{1+y} \in R$ such that

$$f\left(\frac{y}{1+y}\right) = \frac{\left(\frac{y}{1+y}\right)}{1 + \left|\frac{y}{1+y}\right|}$$

$$= \frac{\frac{y}{1+y}}{1\left(\frac{-y}{1+y}\right)} = \frac{y}{1+y-y} = y$$

Let y be positive then $x = \frac{y}{1-y} \in R$ such that

$$f\left(\frac{y}{1-y}\right) = \frac{\left(\frac{y}{1-y}\right)}{1 + \left(\frac{y}{1-y}\right)}$$

$$= \frac{\left(\frac{y}{1-y}\right)}{1 + \left(\frac{-y}{1-y}\right)} = \frac{y}{1-y+y} = y$$

∴ Function f is onto.

Hence, given function f is both one–one and onto.

2.

Show that the function f : R → R given by $f(x) = x^3$ is injective.

Ans - Given function f : R → R given by $f(x) = x^3$.

For one – one:

$f(x) = f(y)$ where $x, y \in R$.

$\Rightarrow x^3 = y^3$ ...... (1)

We need to show that $x = y$. Lets assume $x \neq y$, then,

$\Rightarrow x^3 \neq y^3$

Since this contradicts (1), we get $x = y$.

Hence, function f is injective.

3.

Given a non empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows: For subsets A, B in P(X), ARB if and only if $A \subset B$. Is R an equivalence relation on P(X)? Justify your answer.

Ans - If $A \in P(X)$, we know that every set is a subset of itself, i.e. ARA

⇒ We conclude that R is reflexive.

Let ARB

$\Rightarrow A \subset B$.

This does not mean that $B \subset A$.

If A = {1, 2} and B = {1, 2, 3}, then it cannot be implied that B is related to A.

⇒ We conclude that R is not symmetric.

If ARB and BRC, then $A \subset B$ and $B \subset C$

$\Rightarrow A \subset C$

⇒ ARC

⇒ We conclude that R is transitive.

Hence, R is not an equivalence relation as it is not symmetric.

4.

Find the number of all onto functions from the set {1, 2, 3,......, n} to itself.

Ans - The total number of onto maps from the set {1, 2, 3, ..., n} to itself is equivalent to the total number of permutations of n symbols, namely 1, 2, 3, ..., n.

Since the total number of permutations on n symbols 1,2,3,...,n is n, the total number of onto maps from {1,2,3,...,n} to itself is also n.

5.

Let A $= \{-1, 0, 1, 2\}$, B $= \{-4, -2, 0, 2\}$ and $f, g: A \to B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$. Are f and g equal? Justify your answer.

(Hint: One may note that two function f: A → B and g: A → B such that $f(a) = g(a)\ \forall\ a \in$ A are called equal functions).

Ans - $f(-1) = (-1)^2 - (-1) = 1 + 1$

$\Rightarrow f(-1) = 2$

$g(-1) = 2\left|(-1) - \frac{1}{2}\right| - 1$

$= 2\left(\frac{3}{2}\right) - 1 = 3 - 1$

$g(-1) = 2$

$\Rightarrow f(-1) = g(-1)$

$f(0) = (0)^2 - (0) = 0$

$g(0) = 2\left|(0) - \frac{1}{2}\right| - 1$

$=1 - 1 = 0$

$\Rightarrow f(0) = g(0)$

$f(1) = (1)^2 - (1)$

$= 1 - 1 = 0$

$g(1) = 2\left|(1) - \frac{1}{2}\right| - 1$

$= 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0$

$\Rightarrow f(1) = g(1)$

$f(2) = (2)^2 - (2)$

$= 4 - 2 = 2$

$g(2) = 2\left|(2) - \frac{1}{2}\right| - 1$

$= 2\left(\frac{3}{2}\right) - 1 = 3 - 1 = 2$

$\Rightarrow f(2) = g(2)$

$\therefore f(a) = g(a)\ \forall\ a \in A$. Hence the functions f and g are equal.

6.

Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

(A) 1 (B) 2 (C) 3 (D) 4

Ans - Given a set A = {1, 2, 3}.

Let us take relation R, containing (1, 2) and (1, 3), as R = {(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (3, 1)}.

$(1, 1), (2, 2), (3, 3) \in R$

$\Rightarrow$ We conclude that R is reflexive.

$(1, 2), (1, 3), (2, 1) \in R$

$\Rightarrow$ We conclude that R is symmetric.

$(1, 2), (3, 1) \in R$, but $(3, 2) \notin R$.

$\Rightarrow$ We conclude that R is not transitive.

Relation R will become transitive on adding two pairs (3, 2), (2, 3).

Therefore the total number of desired relations is one.

Hence correct answer is option (A).

7.

Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is

(A) 1 (B) 2 (C) 3 (D) 4

Ans - We are given a set A={1, 2, 3}.

Let us take the relation R, containing (1, 2) as
R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}.

Now the pairs left are (2, 3), (3, 2), (1, 3), (3, 1)

To include the pair (2, 3), it is necessary to also include (3, 2) to maintain symmetry. We must include (1, 3) and (3, 1) to ensure transitivity.

So, only the equivalence relation (bigger than R) is the universal relation.

Therefore, the total number of equivalence relations containing (1, 2) are two.

Hence, correct answer is option (B).